Fluctuating Paths and Fields, Eds. W. Janke, A. Pelster, H.-J. Schmidt, and M. Bachmann (World Scientific, Singapore, 2001).

THE ULTRAVIOLET FIXED-POINT IN QUANTUM ELECTRODYNAMICS – ADLER CONJECTURE: IS QED TRIVIAL?

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We review the question of triviality of QED due to Landau and recall the arguments of the Adler conjecture on the vanishing of the Callan-Symanzik function at the physical fine structure constant, $\beta(\alpha) = 0$.

As a result of the screening of charged particles by their interactions with virtual fermion-antifermion pairs in the vacuum state it is conceivable that there exists no interacting continuum limit of QED in 4 dimensions, a property called "triviality of the theory" by mathematical physicists. From the Renormalization Group (RG) point of view, triviality is a reflection of the *absence* of a nontrivial ultra-violet (UV) stable fixed-point in the Callan-Symanzik beta function. It has been emphasized by Wilson [1] that a nontrivial renormalizable theory can only be formulated, if the Callan-Symanzik function exhibits UV stable fixed-points at a nonzero coupling strength: Only such a fixed-point allows for a *nontrivial continuum limit* of the theory. This was demonstrated in the classic 1954 paper of Gell-Mann and Low [2]. Landau and Pomeranchuk [3] argued that QED is expected to be a *free* theory in this limit. Landau's contention was recently substantiated by lattice calculations [4]. We are now pretty confident that, at infinite cutoff, perturbative QED suffers from complete screening and would have a vanishing fine structure constant. This is somewhat ironic since perturbative renormalization scores its greatest "triumph" in QED!

Nevertheless it is still conceivable that there does exist an UV stable fixed-point, after all, as first argued in the premise in the classic papers of

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Adler [5], and Johnson and Baker [6], although subsequent investigation by Adler, Callan, Gross, and Jackiw and by Baker and Johnson [7] inclined again towards the *absence* of a UV fixed-point. This result was reemphasized by the analysis of Acharya and Narayana Swamy [7].

An UV fixed-point would only be possible, if non-perturbative effects changed the qualitative nature of the operator product expansion, or if there were a non-perturbative renormalization of the triangle anomaly [8]. In this note, we present once more an argument that the Callan-Symanzik beta function $\beta(\alpha)$ does have a nontrivial zero in QED [9], implying a nontrivial continuum limit.

Since the vacuum expectation value of the vector current $\mathcal{J}^\mu$ vanishes by Lorentz invariance, the charge Q must annihilate the vacuum state $|0\rangle$:

$$Q\,|0\rangle = 0. \tag{1}$$

The conservation of the vector current j^μ

$$\partial^\mu j_\mu\,(\mathbf{x},t) = 0 \tag{2}$$

implies the *local* commutator

$$[Q, H\,(\mathbf{x},t)] = 0. \tag{3}$$

The only assumption is that the surface terms at spatial infinity can be discarded. This is justified as long as there are *no* scalar Nambu-Goldstone bosons which could produce a long-range interaction necessary for a non-vanishing surface term at spatial infinity. In QED this is supposed to be the case.

Massless QED is scale-invariant at the classical level. After quantization, the divergence of the dilatation current D_μ is determined by the trace anomaly

$$\partial^\mu D_\mu = \frac{\beta(\alpha)}{\alpha} F_{\mu\nu} F^{\mu\nu}. \tag{4}$$

The dilatation charge

$$Q_D = \int \mathrm{d}^3 x\, D_0\,(\mathbf{x},t) \tag{5}$$

satisfies the commutator relation

$$[Q_D, Q] = -i d_Q Q \tag{6}$$

Fluctuating Paths and Fields, Eds. W. Janke, A. Pelster, H.-J. Schmidt, and M. Bachmann (World Scientific, Singapore, 2001).

which defines the scale dimension of the charge Q, whose canonical value is $d_Q = 0$. We derive from Eqs. (3) and (6), invoking the Jacobi identity, the double commutator

$$[Q_D, [Q, H]] = 0, \tag{7}$$

and arrive at

$$[Q, \partial^\mu D_\mu] = 0. \tag{8}$$

The last step follows from

$$[H(\mathbf{x}, t), Q_D] = -i\partial_\mu D^\mu \neq 0 \tag{9}$$

by virtue of the trace anomaly (4).

Applying (8) to the vacuum state, we obtain

$$[Q, \partial^\mu D_\mu] |0\rangle = 0. \tag{10}$$

Using Eq. (1), this implies

$$Q\partial^\mu D_\mu |0\rangle = 0. \tag{11}$$

The divergence of the dilatation current is clearly a *local* operator. Moreover, the charge Q is *time-independent* (since the vector current is conserved) and has the following important properties:

(a) It is a *constant* operator,

(b) it is also a generator of pure phase rotations in the electron fields.

Therefore, the locality of $\partial^\mu D_\mu$ remains undisturbed by the multiplication with Q. We may thus invoke the Federbush-Johnson theorem [10] which applies to *any* local operator to conclude that

$$Q\partial^\mu D_\mu \equiv 0. \tag{12}$$

Since the charge Q must clearly be *non-vanishing* in QED, we obtain

$$\partial^\mu D_\mu \equiv 0. \tag{13}$$

Together with Eq. (4) this implies the result we wanted to prove:

$$\beta(\alpha) = 0. \tag{14}$$

This is the conclusion first drawn in the classic paper of Adler [5]. It remains an interesting problem to calculate $\beta(\alpha)$ non-perturbatively and verify

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whether that (14) is true or not, and to find out the possible source of error in the previous simple line of arguments.

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This brief essay is offered to celebrate the 60^{th} birthday of Professor Hagen Kleinert (still youthful in appearance and "just smashing"). I have fond memories of his vitality, direct candor, and his innate brilliance. He was most gracious to give me an opportunity to come to Berlin in 1974 and 1975. On that occasion, I had many pleasant discussions with him on this unresolved problem and many other subjects.

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