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FROM Z OPERATOR TO $SO(10)$, NEUTRINO OSCILLATIONS, AND FERMI-DIRAC FUNCTIONS FOR QUARK PARTON DISTRIBUTIONS

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The oscillations advocated to explain the anomalies in solar and atmospheric neutrinos support $SO(10)$ gauge unification. In fact, within reasonable assumptions the highest matrix element of the Majorana mass matrix of right-handed neutrinos has a value in good agreement with the scale of B-L symmetry breaking in the $SO(10)$ theory with Pati-Salam intermediate symmetry. The present experimental knowledge on deep inelastic scattering supports an important role of the Pauli principle, which gives rise to for the correlation between the value of the second moment of each parton and the shape of its distribution.

1 $SO(10)$ and Neutrino Oscillations

Thirty years ago I met Hagen Kleinert at CERN and the very stimulating interaction with him and Carlos Alberto Savoy led our "Florentine" group to a breakthrough [1] in our search for the transformation from constituent to current quarks. Our results turned out to be in very good agreement with the previous phenomenological findings [2] on the chiral content of the baryon octet and on the mixing of the lower meson states.

Some years later Hagen invited me to give lectures in Berlin on the properties of the exceptional algebras coming from the underlying role of octonions [3]. This has been the starting point of my research on $SO(10)$, which can be easily found from $E(6)$ by considering 2×2 rather than 3×3 complex octonionic matrices [4].

Another German physicist, Christoph Wetterich, triggered my attention on the interesting phenomenological properties of $SO(10)$ [5]. Indeed Georgi

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had discovered $SO(10)$ [6] before $SU(5)$ [7], but considered $SU(5)$ more appealing for its peculiar chiral properties [8].

The increasing precision for evaluating the gauge couplings and the lower limit on the proton lifetime excluded minimal $SU(5)$ [9] and asked at least for a modification, as its supersymmetric extension, which leads to a higher unification scale and to a value of α_s slightly larger than the experimental value.

An intriguing aspect of $SO(10)$ has been soon realized: the possibility to get massive neutrinos hierarchically lighter than the other fermions, through the see-saw mechanism [10].

By constructing positive invariants, which vanish for symmetry reasons in certain directions [11], we have been able to build a $SO(10)$ theory with $SU(4) \times SU(2) \times SU(2)$ intermediate symmetry [12] and to conclude [13] that the most promising signature of the unification might be solar neutrino oscillations.

At that time the only hint for that phenomenon came from the Homestake experiment [14], performed as proposed several years before by Bruno Pontecorvo [15], the inventor of neutrino oscillations [16].

We soon realized that the increase of the lower bound on the proton lifetime and the values of the gauge couplings would imply right-handed neutrino Majorana masses at the order of magnitude required by that phenomenon [17]. Later, a reduction of the neutrino flux has been found in the radiochemical GALLEX and SAGE experiments [18], as well as by measuring ν-e scattering at Kamiokande [19]. Finally strong evidence for ν_μ-ν_τ oscillation came from the study of atmospheric neutrinos with a square mass difference around 3.5×10^{-3} $(\mathrm{eV})^2$ and maximal mixing [20].

In presence of several solutions with large mixing angles for solar neutrino oscillations [21], schemes with bimaximal mixing have been proposed either directly [22] for left-handed neutrinos or in the framework of see-saw models [23]: for the latter values of the matrix elements of the Majorana mass M^R of the right-handed neutrinos around 10^{10} - 10^{12} GeV have been found.

We proposed a model [24] with a diagonal Dirac neutrino mass and vanishing diagonal matrix elements for M^R. As in Ref. [23] we found a negligible ν_e content in the heaviest ν^L, almost maximal mixing angle for solar neutrino oscillations and almost opposite eigenvalues for the mass of the two lightest neutrinos.

The vanishing of the 3×3 matrix element, which is a common feature

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of Refs. [23,24], follows from the requirement of a not too large range for the non-vanishing matrix elements of M^R with a maximal mixing angle for $\nu_\mu - \nu_\tau$ and a diagonal Dirac mass matrix [25]. The highest matrix element of M^R, i.e. M^R_{23}, is around 7.5×10^{11} GeV. This is in good agreement with the scale of the symmetry breaking of B-L, 2.8×10^{11}, found in the theory with Pati-Salam intermediate symmetry [26].

We may conclude that the present evidence on neutrino oscillations is a good hint for $SO(10)$ gauge unification [27].

2 Pauli Principle and Parton Distributions

At the beginning of the 1970's, Murray Gell-Mann gave a seminar in Rome, discussing, apart from other topics, the importance of finding the transformation from constituent to current quarks. At the end of the talk I told him that we had found such a transformation some years ago at CERN [1]. He pointed out to us that we had found only a part of the transformation, which should also account for the presence of $q\bar{q}$ pairs, as seen in deep inelastic phenomena.

Our present knowledge is that the result found in Ref. [1] is the right description at $Q^2{=}0$, where baryons are a qqq state. At large Q^2, protons and neutrons, the targets of deep inelastic phenomena, appear as continuous distributions of partons, including $\bar{q}$'s and gluons.

Richard Feynman, the father of path integrals, stated in a work with Field [28], when referring to the proton: "... the pairs $u\bar{u}$ expected to occur in the small x region (the "sea") are suppressed more than $d\bar{d}$ by the exclusion principle."

Despite a theoretical counterargument, this conjecture, as it happened often to Feynman, has been experimentally confirmed by the defect [29] in the Gottfried sum rule [30] and by the sign of the asymmetry found in the Drell-Yan production of muon pairs in pp and pn reactions [31]. Feynman and Field [28] assumed a different high-x behavior for u and d partons in the proton to comply with the dramatic fall at large x of the ratio $F_2^n(x)/F_p^2(x)$ [32]. This different behavior may be also a consequence of the Pauli principle, which may demand broader x-distributions for higher first moments.

This line of thought led Jacques Soffer and myself first to predict [33] the proportionality for $xg_1(x)$ and $F_2^p(x){-}F_2^n(x)$, which holds for the contribution of the $u^\uparrow$ parton in the region dominated by valence quarks. This is the one with the highest first moment, expected to dominate at high x. It should hold for the other valence quarks, if $2u^\uparrow(x) = d(x)$, which is an approximately

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good relation for their first moments. In fact the dominance of $u^\uparrow$ at high x explains both the behaviors of F_n^2/F_2^p and of $g_1^p(x)/F_1^p(x)$ [34], towards 1/4 and 1, respectively, as $x \to 1$.

After describing a large set of deep inelastic data with Fermi-Dirac functions for quark partons [35], a successful test for this assumption has been given in Ref. [36], by showing that the ratio between the third and the second moment of the valence partons is an increasing function of the second moment, as expected by the Pauli principle. We also found a higher second moment for s than for $\bar{s}$ [37], as previously advocated [38].

The consequences of the Pauli principle should disappear at higher Q^2 by the presence of the transverse degrees of freedom. This would imply a large dilution, the Boltzmann limit, where the shapes of the parton distribution would be independent of the first moments. However transverse degrees of freedom for partons of finite longitudinal momentum imply an additional energy, which may limit their role. The evolution equations [39] should be modified by the effect of Pauli blocking for the quarks and of the stimulated emission for gluons [40].

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I am very happy to contribute to the book dedicated to Hagen, which gave me the occasion to recall the good old times of our collaboration and of time spent with with him and Annemarie. It was not only very effective, but also very pleasant, since he is a personality rich of humour and enthusiasm, someone able to transmit these qualities to others. Bridging our national characters, his extrovert character influenced some shy aspects of mine, with the consequence of inducing a better confidence in myself which helped me in facing the challenging topics here described.

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