Fluctuating Paths and Fields, Eds. W. Janke, A. Pelster, H.-J. Schmidt, and M. Bachmann (World Scientific, Singapore, 2001).

STRING, SCALAR FIELD, AND TORSION INTERACTIONS

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It is shown how the scalar field is a necessary part of torsion. Its effects of being incorporated into the torsion tensor are examined.

1 Introduction

Before a great dam collapses, small rivulets form deep holes within its mass, allowing water to trickle through, and then gather enough momentum to erode the mighty structure and reduce it to rubble. Like the water and air separated by the dam, general relativity and quantum mechanics have been kept apart, and despite the countless years and desperate attempts to bring them together, the great wall of ignorance, stronger than any concrete, has held as firm barrier between the two. In recent years, however, we may have witnessed the first trickle cutting through the dam. These tiny rivulets are strings, and if they succeed in gathering sufficient momentum, they may carve out the new river bed for a quantum theory of gravity. It is a pleasure to write this article for the occasion of Hagen Kleinert's sixtieth birthday, whose prolific work in so many areas has surely worn down the barrier, and has established many inroads to developing a more complete picture of physics.

When it was discovered that gravity was incorporated in string theory the landscape began to change in several ways. The string theorists welcomed its honored guest with open arms, and argued that its natural inclusion indicates that string theory is a physical theory, and not simply ornate mathematics. However, string theory did not only invite the gravitational field on stage,

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but an antisymmetric and a scalar field made their appearance as well, and could not be denied by their audience. Scherk and Schwarz [1] made a major breakthrough, creating the first rivulet, showing that the low energy effective Lagrangian of string theory is simply the curvature scalar R of space-time with torsion. This amazing result used more than string to tie together hitherto disparate theories. It gave a rich and compelling low energy Lagrangian that announced the presence of gravity quite effectively, while giving a geometrical interpretation to all three fields.

These results were planted decades ago, but their full harvest has still not been reaped. Many general relativists have remained unwilling to consider gravitation with torsion. String theory, aside the ground breaking work of Kibble [2], Hehl [3], and others, who used torsion in formulating a local Poincaré gauge theory, was unable to provide a rich enough soil to host the full growth of these ideas. In recent years, though, the combined theoretical predictions of string theory and general relativity with torsion have produced a synergism that has breathed new life into both the foundation and physical interpretations of the new fields.

One of the recent advances from the gravitational side showed that the torsion arises from intrinsic spin. This was not only known from the local Poincaré approach, but was also verified by explicit calculations of the equations of motion [4]. In fact, the use of general relativity provided for the detailed interaction of intrinsic spin with gravity or spin. In developing the phenomenological source of spin, it was found that an intrinsic vector, ξ^μ, had to be introduced. At first this vector posed as a mysterious stranger, and only pointed to general hints at physical interpretations. For example, it indicated that the source could not be represented by a point particle but must be at least one-dimensional, but the issue was not resolved until the incorporation of a string source [5].

Not only has the antisymmetric field been given a physical interpretation and home, but the scalar field has been adopted as well. Under my original program torsion was defined as $S_{\mu\nu\sigma} = \psi_{[\mu\nu,\sigma]}$. However, when the source was taken to be given by the Dirac Lagrangian, several more interesting results sprouted. One of these is that the low energy limit of the resulting Dirac equation gave the same interaction as the phenomenological result, and although this shed no light on ξ^μ, it justified its introduction. Another result was the germination of the scalar field. With Dirac coupling, if torsion of the kind given above is included, then a scalar field must be present. In fact, it may be interpreted that the scalar field must arise due to the non-conservation

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of the axial current. In order to incorporate the scalar field in the original seeding, it was shown that the generalization of the torsion to [6]

$$S_{\mu\nu\sigma} = \psi_{[\mu\nu],\sigma} + \frac{2}{3}\epsilon_{\mu\nu\sigma\gamma}\phi^{,\gamma} \tag{1}$$

yields the correct self-consistent results.

With this generalization it was found that the curvature scalar R of gravitation with torsion not only gives the correct low energy effective limit of string theory, it provides a physical interpretation of the origin of the scalar field [7]. It turned out that the source is related to the pseudoscalar invariant, a result that gives a definite direction in which to search for scalar field effects. There was also an improvement over the original results of Ref. [1]: in the current case there are no longer undefined fields, and the covariant derivative of the metric tensor vanishes, which insures constancy of length upon parallel transport.

The ties that bind string theory to gravitation discussed so far, are the antisymmetric field and its definition as torsion, the resulting equivalence of the low energy effective string theory Lagrangian to R, and the fact that gravitation with this torsion would not allow zero-dimensional sources. In recent years the final tie was to formulate gravitation with a string source [8]. This not only provided a natural origin for the antisymmetric field, and gave a very satisfying interpretation to the conservation laws, it provided a physical interpretation of ξ^μ [9]. Shedding its cloak of obscurity, this vector was finally recognized as the tangent vector to the string (see Ref. [9]). In fact, if strings had not been invented already, this line of growth would have led naturally to their doorstep.

Like the nourishment a flood can provide to a barren soil, these new fields plant the possibilities of providing new effects that may be measured. Below, I will establish an interaction between the scalar field and the spin of a particle. This is a new prediction and grows from a very simple foundation. Jumping ahead, the result I will show is that the interaction is given by the scalar product of the spin of a particle and the scalar field gradient.

2 Torsion and the Action

The variation principle is taken to be

$$\delta(I_g + I) = 0, \tag{2}$$

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where

$$I_g = \int \sqrt{-g} \frac{R}{2k} d^4x \tag{3}$$

and I is the material action. The curvature scalar R is that of U_4 space-time. The energy momentum tensor is defined by

$$\delta I_m = \frac{1}{2} \int d^4x \sqrt{-g} T^{\mu\nu} \delta\phi_{\mu\nu}, \tag{4}$$

where $\phi_{\mu\nu} = g_{\mu\nu} + \psi_{\mu\nu}$, which gives an antisymmetric energy momentum tensor that has been discussed in Ref. [9]. The field equations are obtained by taking variations with respect to $\phi_{\mu\nu}$, and are

$$G^{\mu\nu} - 3S^{\mu\nu\sigma}{}_{;\sigma} - 2S^{\mu}{}_{\alpha\beta}S^{\nu\alpha\beta} = kT^{\mu\nu}, \tag{5}$$

where

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \tag{6}$$

and $R^{\mu\nu}$ is the (asymmetric) Ricci tensor in U_4 space-time. The field equations for torsion are given by

$$S^{\mu\nu\sigma}{}_{;\sigma} = -kj^{\mu\nu}, \tag{7}$$

where $j^{\mu\nu} \equiv (1/2)T^{[\mu\nu]}$.

3 The Source, and the Essential Scalar Field

The equations defined above carry little content until $T^{\mu\nu}$ is specified. One should not be swayed by (4), which carries as much content as vacuum: it is merely a definition. All things that are measured come from the equations of motion, or the interaction energies predicted by the theory, but none of these can be obtained until an explicit form of $T^{\mu\nu}$ is given. This is part of the beauty and simplicity of the Einstein equations, or their generalizations, when they are derived from an action principle.

For now, we would like to investigate the effects on the scalar field of using the Dirac Lagrangian as the source. Thus, assume that the material action is given by

$$L = -\frac{i}{2} \left[(D_a \overline{\psi}) \gamma^a \psi - \overline{\psi} \gamma^a D_a \psi - 2im \overline{\psi} \psi \right], \tag{8}$$

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with

$$D_a\psi = \psi_{,a} - \frac{1}{4}\Gamma_{abc}\gamma^b\gamma^c\psi, \tag{9}$$

where Γ_{abc} is the non-holonomic spin connection and contains (1). Thus, we have the variational principle,

$$\delta\int e\left(\frac{R}{2k} + L\right)d^4x = 0, \tag{10}$$

where e is $\sqrt{-g}$.

A comforting surprise reveals itself when R is expanded into ${}^oR$, the curvature scalar of Riemann space-time (no torsion), plus everything else. Ignoring terms that give no contributions to any of the field equations, the action becomes

$$\delta\int\sqrt{-g}d^4x\left(\frac{{}^oR - H_{\mu\nu\sigma}H^{\mu\nu\sigma} + 4\phi_{,\sigma}\,\phi^{,\sigma}}{2k} + L\right) = 0. \tag{11}$$

If we take $k = 8\pi Ge^{2\phi}$, which does not affect the derivation of (11), the low energy effective Lagrangian of string theory results. Our goal is to investigate interactions and possible measurements of the scalar field, so let us consider the weak ϕ limit in the Einstein frame, which yields

$$\delta\int\sqrt{-g}d^4x\left(\frac{{}^oR - S_{\mu\nu\sigma}S^{\mu\nu\sigma} - \frac{1}{2}\phi^{,\sigma}\phi_{,\sigma}}{2k_0} + L\right) = 0\,, \tag{12}$$

where $k_o = 8\pi G$ is the standard constant.

This can be generalized somewhat by introducing non-minimal coupling, in which case the undetermined coupling constant κ is introduced (we get minimal coupling results by letting $\kappa = 1$) [10]. Proceeding in this most general case, the Dirac equation, which is obtained by performing variations with respect to $\overline{\psi}$, is

$$\gamma^a\left(\psi_{,a} - \frac{\kappa}{4}S_{abc}\gamma^b\gamma^c\right)\psi + im\psi = 0, \tag{13}$$

where

$$S_{abc} = e^\alpha_a e^\beta_b e^\gamma_c S_{\alpha\beta\gamma} \tag{14}$$

is the torsion in the anholonomic basis.

One result immediately shows itself from (13): the scalar field interactions with the Dirac particle. Since our current mission is to investigate the effects

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of the scalar field, we will ignore the contribution from $H_{\alpha\beta\gamma}$. The Dirac equation may be separated, in the low energy limit, in terms of its "large" and "small" two-spinors, as usual. Calling Ψ the large two-component spinor, the Dirac equation yields

$$i\hbar\frac{\partial}{\partial t}\Psi = +\frac{p^2}{2m}\Psi + \frac{\sqrt{24}\kappa c}{2}\boldsymbol{S}\cdot\boldsymbol{\nabla}\phi, \tag{15}$$

where $\boldsymbol{S}$ is the spin, and cgs units have been restored.

This is part of the main result, and gives the explicit form of the interaction with the scalar field. It shows that the interaction is given by the scalar product of the intrinsic spin and the gradient of the scalar field, but there is more to come. The field equation for the scalar field, obtained by considering variations with respect to ϕ, may be put in the form

$$\Box\phi = -16\pi\sqrt{6}i\frac{Gm}{c^2}\overline{\psi}\gamma_5\psi. \tag{16}$$

4 Conclusion

The last two equations, (15) and (16), constitute the main result. In words, Eq. (16) states that the scalar field arises from the pseudoscalar invariant, a quantity that vanishes for free particles but not in general. Once this is used in order to find ϕ, Eq. (15) can be taken to find the interaction. Thus, I must leave you hanging, a bit. The next steps in this research program are to evaluate Eq. (16) for any systems that seem propitious, and then to calculate the interaction strength. These results will be used to look for such an interaction, or apply the results to bound the non-minimal coupling constant.

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