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RENORMALIZATION GROUP METHOD AND INHOMOGENEOUS UNIVERSE

Y. NAMBU

Department of Physics, Graduate School of Science,
Nagoya University, Nagoya 464-8602, Japan
E-mail: nambu@allegro.phys.nagoya-u.ac.jp

Using the renormalization group method, we improve the first-order solution of the long-wavelength expansion of the Einstein equation and obtain the renormalization group equation. The solution of the renormalization group equation shows that the renormalized metric describes the behavior of the gravitational collapse in the expanding Universe qualitatively well and is suitable for modeling an inhomogeneous Universe.

1 Introduction

Our Universe seems to be very close to a Friedmann-Robertson-Walker (FRW) space-time at a length scale of the order of the Hubble radius, but the metric and matter content appear to be highly inhomogeneous at smaller scales. The conventional cosmological perturbative approach [1] treats such a situation as the homogeneous isotropic background plus a small perturbation, and investigates the evolution of linear fluctuations. We must go beyond the linear approximation to treat the nonlinear structure and to construct a suitable model of an inhomogeneous Universe which is close to a FRW Universe on a large scale. The spatial gradient expansion [2] of the Einstein equation is a nonlinear approximation method which describes the long-wavelength inhomogeneity in the Universe. This approximation scheme expands the Einstein equation with respect to the order of the spatial gradient. As a background solution, we solve Einstein's equation by neglecting all spatial gradient terms. The resulting solution has the same form as that for the spatially flat FRW

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Universe, but the three-metric can have a spatial dependence. It is possible to include the effect of spatial gradient terms by calculating the next order. This method can describe a long-wavelength nonlinear perturbation without imposing any symmetry for a space, and is suitable for analyzing the global structure of an inhomogeneous Universe.

However, this scheme is valid only for a perturbation whose wavelength is larger than the Hubble horizon scale. For a matter field which satisfies the energy conditions, the perturbation terms induced by the spatial gradient terms grow in time and finally dominate the background solution. This occurs when the wavelength of the perturbation equals the Hubble horizon scale. After this time, the wavelength of the perturbation becomes shorter than the horizon scale and the result of the gradient expansion becomes unreliable.

A similar situation occurs in the field of nonlinear dynamical systems. To obtain the temporal evolution of the solution of a nonlinear differential equation, we usually apply a perturbative expansion. But naive perturbation often yields secular terms due to resonance phenomena. The secular terms prevent us from getting approximate but global solutions. There are many techniques to circumvent the problem, for example, the averaging method, the multi-time scale method, the WKB method and so on [3]. Although these methods yield globally valid solutions, they provide no systematic procedure for general dynamical systems because we must select a suitable assumption on the structure of the perturbation series.

The renormalization group method [4] as a tool for a global asymptotic analysis of the solution to differential equations unifies the techniques listed above, and can treat many systems irrespective of their features. Starting from a naive perturbative expansion, the secular divergence is absorbed in the constants of integration contained in the zeroth-order solution by the renormalization procedure. The renormalized constants obey the renormalization group equation. This method can be viewed as a tool of system reduction. The renormalization group equation corresponds to the amplitude equation which describes slow motion dynamics in the original system. We can describe complicated dynamics contained in the original equation by extracting a simpler representation using the renormalization group method.

In this article, we apply the renormalization group method to the gradient expansion of Einstein's equation. Our purpose is to obtain the renormalized long-wavelength solution of Einstein's equation which is also valid for later times. Through the procedure of renormalization, we extract slow motion from the Einstein equation [5].

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2 Renormalization Group Method

The renormalization group method [4] improves the long-time behavior of a naive perturbative expansion. We explain the basic concept of the renormalization group method using two examples. The first one is a harmonic oscillator. The equation of motion is

$$\ddot{x} + x = -\epsilon\, x, \tag{1}$$

where ϵ is a small parameter. We solve this equation perturbatively by expanding the solution with respect to ϵ:

$$x = x_0 + \epsilon\, x_1 + \cdots\ . \tag{2}$$

The solution up to $O(\epsilon)$ becomes

$$x = B_0 \cos t + C_0 \sin t + \frac{\epsilon}{2}\,(t - t_0)(C_0 \cos t - B_0 \sin t) + O(\epsilon^2), \tag{3}$$

where B_0 and C_0 are constants of integration determined by the initial condition at arbitrary time $t = t_0$. This naive perturbation breaks down when $\epsilon\,(t - t_0) > 1$ because of the secular term. To regularize the perturbation series, we introduce an arbitrary time μ, split $t - t_0$ as $t - \mu + \mu - t_0$, and absorb the divergent term containing $\mu - t_0$ into the renormalized counterparts B and C of B_0 and C_0, respectively.

We introduce renormalized constants as follows:

$$B_0 = B(\mu) + \epsilon\, \delta B(\mu, t_0), \quad C_0 = C(\mu) + \epsilon\, \delta C(\mu, t_0), \tag{4}$$

where δB and δC are counter terms that absorb the terms containing $\mu - t_0$ in a naive solution. Inserting Eq. (4) in Eq. (3), we have

$$\begin{aligned} x = B(\mu) \cos t + C(\mu) \sin t + \epsilon \Big\{ \delta B \cos t + \delta C \sin t \\ + \frac{1}{2}(t - \mu + \mu - t_0)(C(\mu) \cos t - B(\mu) \sin t) \Big\}. \end{aligned} \tag{5}$$

We choose δB and δC as

$$\delta B(\mu, t_0) + \frac{1}{2}(\mu - t_0) C(\mu) = 0, \quad \delta C(\mu, t_0) - \frac{1}{2}(\mu - t_0) B(\mu) = 0. \tag{6}$$

Using the relations $\epsilon\, \delta B = B_0 - B(\mu)$, and $\epsilon\, \delta C = C_0 - C(\mu)$, we obtain

$$B(t_0) = B(\mu) - \frac{\epsilon}{2}(\mu - t_0) C(\mu), \quad C(t_0) = C(\mu) + \frac{\epsilon}{2}(\mu - t_0) B(\mu). \tag{7}$$

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These equations define the transformation up to $O(\epsilon)$:

$$\mathcal{R}_{\mu-t_0}: \; (B(t_0), C(t_0)) \to (B(\mu), C(\mu)),$$

and the transformation forms a Lie group up to $O(\epsilon)$. Assuming the properties of a Lie group, we can extend the locally valid expression (7) to a global one, which is valid for arbitrary large $\mu - t_0$. We apply this transformation to get $(B(\mu), C(\mu))$ at arbitrary large μ. By differentiating Eq. (7) with respect to μ and setting $t_0 = \mu$, we have the renormalization group equations

$$\frac{\partial B}{\partial \mu} = \frac{\epsilon}{2}\, C(\mu), \qquad \frac{\partial C}{\partial \mu} = -\frac{\epsilon}{2}\, B(\mu). \tag{8}$$

The renormalized solution becomes

$$x = B(\mu)\cos t + C(\mu)\sin t + \frac{\epsilon}{2}\,(t-\mu)[C(\mu)\cos t - B(\mu)\sin t]. \tag{9}$$

Solving the renormalization group equations (8) and equating μ and t in (9) eliminates the secular term and we get a uniformly valid result

$$x = B(0)\cos\left(1+\frac{\epsilon}{2}\right)t + C(0)\sin\left(1+\frac{\epsilon}{2}\right)t. \tag{10}$$

The second example is the Einstein equation for a FRW Universe with dust. The spatial component is

$$\ddot{\alpha} + \frac{3}{2}\dot{\alpha}^2 = -\frac{\epsilon}{2}e^{-2\alpha}, \tag{11}$$

where $\alpha(t)$ is the logarithm of the scale factor of the Universe $a(t)$ and ϵ is the sign of the spatial curvature. The exact solution is given by

$$a(t) = e^{\alpha(t)} = \begin{cases} a_0\,(1-\cos\eta), & t = a_0\,(\eta - \sin\eta) \qquad \text{for } \epsilon = 1, \\ a_0\,\eta^2/2, & t = a_0\,\eta^3/6 \qquad \text{for } \epsilon = 0, \\ a_0\,(\cosh\eta - 1), & t = a_0\,(\sinh\eta - \eta) \qquad \text{for } \epsilon = -1. \end{cases} \tag{12}$$

We solve Eq. (11) perturbatively by assuming that the right hand side is small. This represents an expansion with respect to a small spatial curvature around the flat Universe. By substituting $\alpha = \alpha_0 + \epsilon\,\alpha_1 + \cdots$ in Eq. (11), we find the naive solution

$$\alpha = \ln\tau + C_0 - \epsilon\,\frac{9}{20}e^{-2C_0}(\tau - \tau_0) + O(\epsilon^2), \tag{13}$$

where $\tau = t^{2/3}$ is a new time variable and C_0 a constant of integration determined by the initial condition at $\tau = \tau_0$. The $O(\epsilon)$ term is secular and we

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regularize this term by introducing the arbitrary time μ and the renormalized constant $C_0 = C(\mu) + \epsilon\, \delta C(\mu, \tau_0)$:

$$\alpha = \ln \tau + C(\mu) + \epsilon\, \delta C(\mu, \tau_0) - \epsilon\, \frac{9}{20} e^{-2C(\mu)} (\tau - \mu + \mu - \tau_0). \tag{14}$$

The counter term δC is determined in such a way that it absorbs the term depending on $\mu - \tau_0$:

$$\delta C(\mu, \tau_0) - \frac{9}{20} e^{-2C(\mu)} (\mu - \tau_0) = 0. \tag{15}$$

This defines the renormalization group transformation

$$\mathcal{R}_{\mu - \tau_0} : \; C(\tau_0) \; \rightarrow \; C(\mu)$$

according to

$$C(\mu) = C(\tau_0) - \epsilon\, \frac{9}{20} e^{-2C(\mu)} (\mu - \tau_0), \tag{16}$$

and this transformation forms a Lie group up to $O(\epsilon)$. So we can have $C(\mu)$ for arbitrary large values of $\mu - \tau_0$ by assuming the property of a Lie group. This makes it possible to produce a globally uniform approximative solution of the original equation. The renormalization group equation reads

$$\frac{\partial C(\mu)}{\partial \mu} = -\epsilon\, \frac{9}{20} e^{-2C(\mu)}, \tag{17}$$

and its solution is

$$C(\mu) = \frac{1}{2} \ln \left(c - \frac{9\epsilon}{10} \mu \right), \tag{18}$$

where c is a constant of integration. The renormalized scale factor is given by

$$a(t) = e^{\alpha(t)} = \tau\, e^{C(\tau)} = t^{2/3} \left(c - \frac{9\epsilon}{10} t^{2/3} \right)^{1/2}. \tag{19}$$

As the zeroth-order solution, it is possible to include another integration constant t_0 which defines the origin of the cosmic time t. By requiring that the renormalization group transformation forms a Lie group, it can be shown that t_0 is not renormalized. Hence it is sufficient to consider the solution with the boundary condition $a(t = 0) = 0$ which fixes the value of t_0 to zero. This point is different from the example of the harmonic oscillator, in which case two integration constants B and C are renormalized.

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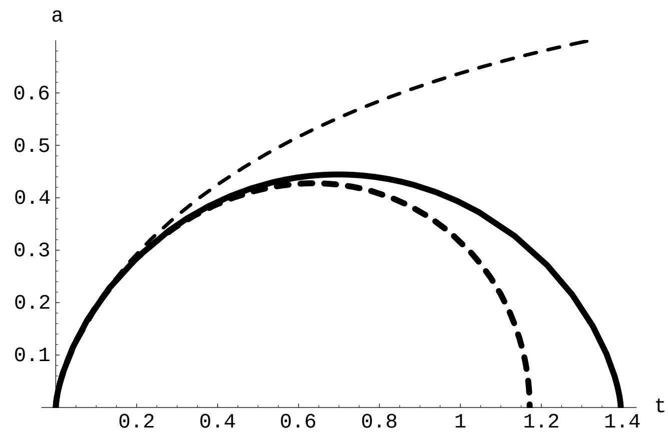

Figure 1. The evolution of the scale factor for a closed FRW Universe with dust. The solid curve is the exact solution, the thin dashed curve is the naive solution and the thick dashed curve is the renormalized solution.

We compare the renormalized solution (19) with the exact solution (12) and the naive solution (13) for the case of a closed Universe ($\epsilon = 1$). We choose $a_0 = 2/9$ and $c = 1$. The scale factor of the exact solution has a maximum at $t = 2\pi/9$ and goes to zero at $t = 4\pi/9$. The naive solution does not show this behavior. The renormalized solution improves the naive solution and reproduces the expanding and contracting feature of the exact solution (Fig. 1).

3 Application of Renormalization Group Method to Gradient Expansion

For the dust dominated Universe, the long-wavelength solution up to the second order of the spatial gradient becomes

$$\begin{aligned}\gamma_{ij} &= t^{4/3}\left[h_{ij} - \frac{9}{5}t^{2/3}\left(R_{ij}(h) - \frac{1}{4}R(h)h_{ij}\right)\right], \\ \rho &= \frac{4}{3t^2}\left(1 - \frac{9}{20}t^{2/3}R(h)\right), \quad u_i = 0,\end{aligned} \tag{20}$$

where h_{ij} is the seed metric which is an arbitrary function of the spatial coordinate. We can see that the perturbation term, which originated from the spatial gradient of the seed metric h_{ij}, grows as the Universe expands

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and finally has the same amplitude as the background term at $t \sim H^{-1}$ when the wavelength of the perturbation equals the Hubble horizon scale. After this time, the wavelength of perturbation becomes smaller than the horizon scale and the long-wavelength expansion breaks down. To make the gradient expansion applicable to the perturbation whose wavelength is smaller than the horizon scale, we use the renormalization group method.

We renormalize the secular behavior of the three-metric γ_{ij}. By introducing a new time variable $\tau = t^{2/3}$ and the initial time τ_0 by redefining the seed metric h_{ij}, we define the renormalized metric and the counter term $h_{ij}(x) = h_{ij}(x,\mu) + \delta h_{ij}(x,\mu,\tau_0)$ as

$$\begin{aligned} h_{ij}(x) &- \frac{9}{5}(\tau-\tau_0)\left(R_{ij} - \frac{1}{4}R(h)h_{ij}\right) \\ &= h_{ij}(x,\mu) + \delta h_{ij} - \frac{9}{5}(\tau-\mu+\mu-\tau_0)\left(R_{ij} - \frac{1}{4}R(h)h_{ij}\right). \end{aligned} \tag{21}$$

By determining the counter term in such a way that it absorbs terms containing $\mu - \tau_0$, we have

$$h_{ij}(x,\mu) = h_{ij}(x,\tau_0) - \frac{9}{5}(\mu-\tau_0)\left(R_{ij}(h(\mu)) - \frac{1}{4}R(h(\mu))h_{ij}(\mu)\right). \tag{22}$$

This equation defines the renormalization group transformation

$$\mathcal{R}_{\mu-\tau_0} : h_{ij}(\tau_0) \to h_{ij}(\mu),$$

which is a Lie group up to $O(\epsilon)$. We can therefore get the value of $h_{ij}(\mu)$ for arbitrary μ using relation (22) by assuming the property of a Lie group. The renormalization group equation is obtained by differentiating Eq. (22) with respect to μ and setting $\tau_0 = \mu$,

$$\frac{\partial}{\partial\mu}h_{ij}(x,\mu) = -\frac{9}{5}\left[R_{ij}(h(\mu)) - \frac{1}{4}R(h(\mu))h_{ij}(\mu)\right], \tag{23}$$

such that the renormalized solution is

$$\gamma_{ij} = t^{4/3}h_{ij}(x,t), \quad \rho = \frac{4}{3t^2} + \frac{3t^{-4/3}}{10}R(h(x,t)). \tag{24}$$

We solve the renormalization group equation (23) for some special cases and see how the renormalization group method improves the behavior of the long-wavelength solution.

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3.1 FRW Case

The metric is

$$h_{ij}(x,t) = \Omega^2(t)\,\sigma_{ij}(x), \quad R_{ij}(\sigma) = \frac{1}{3}\sigma_{ij}\,R(\sigma), \tag{25}$$

where $\sigma_{ij}(x)$ is the metric of the three-dimensional maximally symmetric space. In this case, the renormalization group equation (23) reduces to

$$\frac{\partial}{\partial\tau}\Omega^2(\tau) = -\frac{9}{10}k, \quad (k = \pm 1, 0), \tag{26}$$

and the renormalized solution is

$$\gamma_{ij} = t^{4/3}\left(c - \frac{9k}{10}\,t^{2/3}\right)\sigma_{ij}(x), \quad \rho = \frac{4}{3t^2}\left(1 + \frac{27}{20}\frac{k\,t^{2/3}}{c - \frac{9k}{10}\,t^{2/3}}\right). \tag{27}$$

Thus the scale factor of the Universe is given by

$$a(t) = t^{2/3}\sqrt{c - \frac{9k}{10}t^{2/3}}, \tag{28}$$

which is the same as the solution (19).

3.2 Spherically Symmetric Case

In spherical coordinates (r, θ, ϕ), the metric reads

$$h_{ij} = \mathrm{diag}\left(A^2(\tau,r), B^2(\tau,r), B^2(\tau,r)\sin^2\phi\right). \tag{29}$$

The renormalization group equation (23) becomes for the (rr component) and the ($\theta\theta$ component)

$$2A\frac{\partial A}{\partial\tau} = \frac{9}{5}\left(\frac{A^2}{2B^2} - \frac{A_{,r}B_{,r}}{AB} - \frac{(B_{,r})^2}{2B^2} + \frac{B_{,rr}}{B}\right), \tag{30}$$

$$2B\frac{\partial B}{\partial\tau} = \frac{9}{5}\left(-\frac{1}{2} + \frac{(B_{,r})^2}{2A^2}\right), \tag{31}$$

respectively. The solution is given by

$$B = \left(1 - \frac{9\alpha(r)}{10}\tau\right)^{1/2}\beta(r), \quad A = \frac{B_{,r}}{\sqrt{1 - \alpha(r)\beta^2(r)}}, \tag{32}$$

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where α and β are arbitrary functions of r. The renormalized metric and density are

$$ds^2 = -dt^2 + \frac{\left(t^{2/3}B_{,r}\right)^2}{1-\alpha\,\beta^2}\,dr^2 + \left(t^{2/3}B\right)^2 d\Omega_2^2, \tag{33}$$

$$\rho = \frac{4}{3t^2} + \frac{3}{5t^{4/3}} \frac{3\alpha\,\beta_{,r}\left(1-\frac{9\alpha}{10}\tau\right) + \alpha_{,r}\,\beta\,\left(1-\frac{27\alpha}{20}\tau\right)}{\left(1-\frac{9\alpha}{10}\tau\right)\left[\left(1-\frac{9\alpha}{10}\tau\right)\beta_{,r} - \frac{9}{20}\alpha_{,r}\,\beta\,\tau\right]}. \tag{34}$$

This solution corresponds to the Toleman-Bondi solution [6]. The renormalized solution reproduces very well the feature of the metric of the spherically symmetric gravitational collapse of dust.

3.3 Szekeres Solution

In Cartesian coordinates (x, y, z), the metric is assumed to be

$$h_{ij} = \mathrm{diag}\left(1, 1, A^2(\tau, x, y, z)\right). \tag{35}$$

The renormalization group equation (23) reduces to the following three equations for the xy, xx, and zz component

$$\begin{aligned} 0 &= -\frac{A_{,xy}}{A}, \\ 0 &= \frac{A_{,yy} - A_{,xx}}{2A}, \\ \frac{\partial A^2}{\partial \tau} &= -\frac{1}{2}A(A_{,xx} + A_{,yy}). \end{aligned} \tag{36}$$

The renormalized metric is

$$ds^2 = -dt^2 + t^{4/3}\left[dx^2 + dy^2 + \left(g(z)\left(x^2+y^2\right) + \frac{9g(z)}{5}t^{2/3} + c(z)\right)^2 dz^2\right]. \tag{37}$$

This is the exact solution [7] of Szekeres, which represents a one-dimensional gravitational collapse. It is known that the "naive" gradient expansion reproduces this solution by including the fourth-order spatial gradient [2]. We obtained the solution using the second-order spatial gradient with renormalization. In this case, the renormalization procedure strongly improves the naive solution.

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4 Summary

After applying the renormalization group method to improve the long-time behavior of the solution of the gradient expansion, we obtained the solutions of the renormalization group equation for FRW, spherically symmetric and Szekeres cases. The behavior of the renormalized solution indicates that they describe the collapsing phase of the system qualitatively well. The renormalization group method is regarded as the procedure of system reduction. This means the renormalization group Eq. (23) is the reduced version of the original Einstein equation and describes the slow motion dynamics of the original equation. We expect that the renormalization group equation (23) has physically interesting properties and solutions which are contained in the original Einstein equation.

We can consider the cosmological back reaction problem [8,9] from the point of view of the renormalization of the fluctuation. The naive solution represents the evolution of the perturbation with the fixed background metric. By renormalizing the naive solution, the constants contained in the background solution become time dependent due to the spatial inhomogeneity. Therefore we can investigate how the spatial inhomogeneity affects the "background" metric by solving the renormalization group equation. Based on the conventional cosmological perturbation, we can describe the back reaction effect by renormalizing the second-order zero mode perturbation [10].

For the quantum dynamics of an inflationary Universe, it is possible to derive the basic equation of the stochastic approach by using the renormalization group method. In this case, the long-wavelength quantum fluctuation is renormalized to the slowly varying background classical field. We believe that the renormalization group method gives us further understanding of the inhomogeneous cosmology.

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