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SCALING AND DUALITY IN THE SUPERCONDUCTING PHASE TRANSITION

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The field theoretical approach to duality in the superconducting phase transition is reviewed. Emphasis is given to the scaling behavior, and recent results are discussed.

1 Introduction

The renewed interest for critical fluctuations in superconductors is due to the enormous variety of interesting phenomena observed in the last decade. In a zero external field regime, non-classical values of the critical exponents and amplitude ratios were measured [1]. In the observed critical region we have $-0.03 < \alpha < 0$, $\nu \approx 0.67$ and $A_+/A_- \approx 1.065$. These values were measured for bulk samples of $YBa_2Cu_3O_{7-\delta}$ (YBCO). They indicate that the observed critical region corresponds to the XY universality class. In the non-zero field regime, the thermal fluctuations make the vortex lattice melt. For high fields this transition is known to be of first order [2].

Most thermal fluctuation effects in superconductors can be understood using the Ginzburg-Landau (GL) model. In zero field, the GL model is given by

$$L = \frac{1}{2}(\nabla \times \mathbf{A})^2 + |(\nabla - ie\mathbf{A})\phi|^2 + m^2|\phi|^2 + \frac{u}{2}|\phi|^4. \tag{1}$$

The different regimes of the GL model are controlled by the size of the Ginzburg parameter $\kappa = \sqrt{u/2e^2}$. In the weak-coupling regime ($\kappa \ll 1$) the

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critical fluctuations in the GL model lead to a fluctuation induced first-order phase transition [3]. This scenario no longer holds in the strong-coupling regime ($\kappa \gg 1$), where a second-order phase transition takes place [4,5]. Usually it is very difficult to access the strong-coupling regime of the GL model through conventional perturbative methods. An alternative approach uses duality arguments. Duality allows us to transform a strong-coupling problem into a weak-coupling one. A well-known example is the 2d Ising model where such a transformation has an additional feature: Two-dimensionality makes the model self-dual, which leads to an exact determination of the critical temperature [6]. In three dimensions life is more complicated, but duality remains a powerful tool. In the case of the GL model, lattice duality studies [4] helped to conclude that the transition must be of second order in the strong-coupling regime. A deeper point of view was pioneered by Kleinert who developed a scaling (continuum) limit of the lattice dual model [5,7,8] which gives a field theoretic description of vortex lines. Using this field theoretical approach, Kleinert made the remarkable discovery that a tricritical point exists in the phase diagram of a superconductor. The tricritical point separates the first- and second-order phase transition regimes of the superconductor. We shall see in the next section that this discovery has far reaching consequences and is useful even in a nonzero field regime.

In zero field, the vortex lines are closed loops, and the corresponding field theory features a *disorder parameter* field ψ (as opposed to the *order parameter* field ϕ). The field ψ describes a grand canonical ensemble for vortex loops, and $|\psi|^2$ gives the vortex density. The duality transformation has transformed a field theory, where the basic objects are the Cooper pairs, into another one, where the basic objects are vortex lines. In the case of the GL model, currents interact through the eletromagnetic vector potential $\mathbf{A}$, while in the disorder field theory they interact through a fluctuating field which is proportional to the *magnetic induction* field $\mathbf{h}$. The simplest example of duality is obtained in the London limit, where the amplitude fluctuations are frozen. There, the dual Lagrangian corresponds to the London model:

$$L_{\rm London}^{\rm dual} = \frac{1}{2}(\nabla\times\mathbf{h})^2 + \frac{m_A^2}{2}\mathbf{h}^2 + im_A\mathbf{J}_v\cdot\mathbf{h}, \tag{2}$$

where m_A is the photon mass and $\mathbf{J}_v$ is the vortex current. The full disorder field theory corresponds to a generalization of this model. Intuitively this can be done as follows: in the classical limit, thermal fluctuations are absent and, therefore, there are no vortex loops. The only way to create vortices

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is by applying an external magnetic field. This creates vortex lines parallel to this field but no loops. The classical solution is well known in this case and corresponds to the Abrikosov vortex lattice [9]. In the context of the model (2), the external magnetic field couples linearly to the induction field $\mathbf{h}$. Thermal fluctuations create additional vortex loops. They are closed as a consequence of Ampere's law which gives $\nabla \cdot \mathbf{J}_v = 0$. In the disorder field theory of fluctuating vortex loops, the coupling $\mathbf{J}_v \cdot \mathbf{h}$ in (2) becomes a minimal coupling, and the result is Kleinert's dual Lagrangian [7,8]

$$L_d = \frac{1}{2}[(\nabla \times \mathbf{h})^2 + m_A^2 \mathbf{h}^2] + |(\nabla - ie_d\mathbf{h})\psi|^2 + m_\psi^2|\psi|^2 + \frac{u_\psi}{2}|\psi|^4, \quad (3)$$

where $e_d = 2\pi m_A/e$ is the dual charge.

In the next sections we shall review recent results on Kleinert's model. We shall discuss the relation between the tricritical point as obtained from the dual model and the tricritical point in the original GL model. It will be shown that the tricritical point in the GL model is of a Lifshitz type [12]. In the GL model, the most important manifestation of the Lifshitz point is a negative sign of the anomalous dimension of the order parameter. Duality transforms the tricritical Lifshitz point into an ordinary tricritical point. As a consequence, the sign of the anomalous dimension of the disorder parameter will be positive.

In Section 3 we discuss the scaling behavior of the dual model, which allows for many possibilities of scaling due to the massiveness of the induction field [13]. Of course, only one scaling corresponds to the superconducting phase transition as obtained from the original GL model, that is, the transition which is governed by an infrared stable charged fixed point. The other scalings correspond to crossover regimes. In principle, all these regimes can be observed experimentally. Ironically, the true superconducting phase transition remains the most difficult to be observed experimentally. In fact, the critical region of this charge fluctuation regime is very small [7]. The regime very often probed is the XY regime [1] and we will see that there is a corresponding scaling in the dual Lagrangian that corresponds to it. A scaling that deserves some experimental attention is the Kleinert scaling [8] which is characterized by the *exact* value $\nu' = 1/2$ of penetration depth exponent. This scaling seems to be verified in two experiments using thin films of YBCO at optimal doping [14,15]. At optimal doping 3D fluctuations are still dominant even in thin films of YBCO and a three-dimensional model is still relevant.

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2 Tricritical and Lifshitz Point

Let us briefly review Kleinert's discovery of the tricritical point. In order to make the discussion simple, we will take advantage of the intuitive point of view adopted in the introduction. The technical details can be found in the textbook of Kleinert [7] and in the seminal paper Ref. [5].

Let us integrate out the induction field $\mathbf{h}$ in Eq. (3). We obtain the following effective action:

$$\begin{aligned} S_{\rm eff} &= \frac{1}{2}\mathrm{Tr}\ \ln[(-\partial^2 + m_A^2 + e_d^2|\psi|^2)\delta_{\mu\nu} + (1-1/a)\partial_\mu\partial_\nu] \\ &+\frac{e_d^2}{2}\int d^3r \int d^3r' j_\mu(\mathbf{r}) D_{\mu\nu}(\mathbf{r},\mathbf{r}') j_\nu(\mathbf{r}') \\ &+\int d^3r \left(|\nabla\psi|^2 + m_\psi^2|\psi|^2 + \frac{u_\psi}{2}|\psi|^4\right), \end{aligned} \tag{4}$$

where j_μ is the μ component of the current operator $\mathbf{j} = ie_d(\psi^\dagger\nabla\psi - \psi\nabla\psi^\dagger)$ and the limit $a \to 0$ must be taken at the end in order to enforce the constraint $\nabla\cdot\mathbf{h} = 0$. The kernel $D_{\mu\nu}(r, r')$ is the inverse of the operator inside the Tr ln. Now we will perform a Landau expansion of the effective action (4). As usual, in this expansion we assume that ψ has no spatial variation. In this way we obtain the following free energy density:

$$\begin{aligned} \mathcal{F} &= [m_\psi^2 + e_d^2 D_{0;\mu\mu}(0)]|\psi|^2 + \frac{1}{2}\left[u_\psi - e_d^4\int\frac{d^3k}{(2\pi)^3}\hat{D}_{0;\mu\nu}(k)\hat{D}_{0;\nu\mu}(k)\right]|\psi|^4 \\ &+\frac{e_d^3}{3}\int\frac{d^3k}{(2\pi)^3}\hat{D}_{0;\mu\lambda}(k)\hat{D}_{0;\lambda\delta}(k)\hat{D}_{0;\delta\mu}(k), \end{aligned} \tag{5}$$

where $D_{0;\mu\nu}(\mathbf{r}-\mathbf{r}')$ is the kernel $D_{\mu\nu}(\mathbf{r},\mathbf{r}')$ for $\psi = 0$ and $\hat{D}_{0;\mu\nu}(k)$ is its Fourier transform which is given by

$$\hat{D}_{0;\mu\nu}(k) = \frac{1}{k^2 + m_A^2}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right). \tag{6}$$

After calculating the integral in the $|\psi|^4$ term in (5), we see that it will be negative if $u_\psi < 4\pi^3 m_A^3/e^4$. When this happens, we have a first-order phase transition scenario. Thus, it is clear that the point $m_\psi^2 = 2\pi m_A^3/e^2$, $u_\psi = 4\pi^3 m_A^3/e^4$ corresponds to a tricritical point (we have redefined m_ψ^2 by absorbing a factor $e_d^2\Lambda/\pi^2$, where Λ is the ultraviolet cutoff).

Now we can ask the following question: How does the tricritical point manifest itself in the GL model? From a RG point of view we have the

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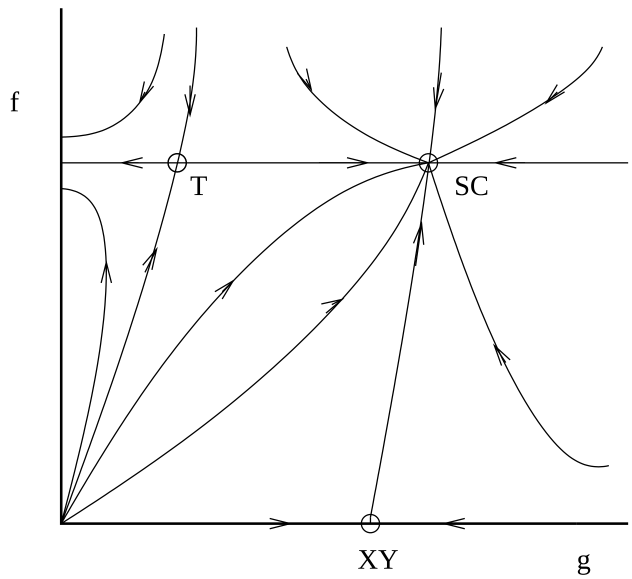

Figure 1. Flow diagram for the GL model. The labels T and SC are for tricritical and superconducting, respectively.

following scenario: let us define the renormalized dimensionless couplings $f = e_r^2/\mu$ and $g = u_r/\mu$, where e_r and u_r are the renormalized couplings and μ is a running scale. The fixed point structure in the gf-plane contains four fixed points [16–19]. Two of them are uncharged: the Gaussian fixed point corresponding to mean-field behavior of an uncharged superfluid, and the XY fixed point that governs the critical behavior of ^{4}He superfluid. The other two fixed points are charged: the infrared stable fixed point that governs the critical behavior of the superconductor and the tricritical fixed point, which is infrared stable along a line connecting the Gaussian and the tricritical fixed point, being unstable in the g-direction. The line connecting the Gaussian fixed point and the tricritical point is called the tricritical line. This line separates the regimes of first- and second-order phase transitions. The schematic flow diagram is shown in Fig. 1. The tricritical point obtained in Kleinert's duality map [5,7] corresponds to the tricritical fixed point of the RG picture. However, this tricritical point obtained directly in the GL model is of a different nature due to the local gauge symmetry. Indeed, we

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have suggested recently [11] that the tricritical point of the GL model is of a Lifshitz type. This picture is founded on the behavior of the 2-point bare correlation function, which is given at 1-loop and $d = 3$ by

$$\tilde{W}^{(2)}(p) = \frac{1}{p^2 + m^2 + \Sigma(p)}, \tag{7}$$

with the self-energy

$$\Sigma(p) = -\frac{m}{2\pi}(u + e^2) - \frac{e^2}{4\pi|p|}(p^2 - m^2)\left[\frac{\pi}{2} + \arctan\left(\frac{p^2 - m^2}{2m|p|}\right)\right]. \tag{8}$$

In writing the above equations, we have absorbed in the bare mass a contribution with a linear dependence on the ultraviolet cutoff Λ. From Eq. (7) we see that $\tilde{W}^{(2)}(p)$ has a real pole in the critical regime ($m^2 = 0$) at a nonzero momentum, besides the usual pole at $p = 0$. In the above 1-loop calculation this pole is at $|p| = e^2/4$. This means that $\tilde{W}^{(2}(p) < 0$ when $|p| < e^2/4$ and the bare 2-point correlation function violates the infrared bound $0 < \tilde{W}^{(2)}(p) \leq 1/p^2$, usually satisfied for pure scalar models [20]. This violation of the infrared bound explains the negative sign of the η exponent usually found in RG calculations [16–19]. The negativeness of the η exponent is also confirmed by recent Monte Carlo calculations [21]. Since the 2-point critical correlation function changes its sign, it follows that the same sign change happens in the 1-particle irreducible 2-point function $\tilde{\Gamma}^{(2)}(p)$, which is the coefficient of the quadratic term in the effective action Γ. This change of sign with momentum at the critical point is a behavior characteristic of a Lifshitz point [12]. It is worth to mention that in scalar models of Lifshitz points the sign of η is also negative for the dimension $d = d_c - 1$ where d_c is the corresponding critical dimension. For instance, a fixed-dimension calculation in a $1/N$ expansion gives, for the isotropic Lifshitz point in $d = 7$ ($d_c = 8$ in this case), $\eta_{l4} \approx -0.08/N$ [22].

The phase transition scenario that emerges is the following. The phase diagram in the $\kappa^2 - T$ plane contains three phases: the normal phase, the type I and the type II regime. The type I regime is separated from the type II regime by a line terminating at a tricritical Lifshitz point, the latter belonging to a line that separates the normal phase from the two other phases. The phase diagram is therefore quite similar to that of the so-called $R - S$ model [23]. In the $R - S$ model the phase diagram is drawn in the $X - T$ plane where $X = S/R$, S and R being the couplings of the model. The three phases of the $R - S$ model are paramagnetic, ferromagnetic and helical.

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Thus, the $R - S$ model differs from ordinary magnets by the presence of a modulated regime for the order parameter, the helical phase. If κ^2 plays a role analogous to X, we see that the type II regime is analogous to the helical phase. Indeed, the type II regime should correspond to a modulated order parameter, as can be seen experimentally by applying a magnetic field, which leads to the formation of the Abrikosov vortex lattice. The type I regime, on the other hand, must be associated to the ferromagnetic phase since it corresponds to a uniform order parameter.

3 Kleinert's Scaling in the Dual Model

Let us discuss the renormalization of the dual model Eq. (3). An important feature of the dual model is the presence of the two mass scales m_ψ and m_A. This fact allows some freedom in the scaling of the model which is clarified in Ref. [13].

The scaling is defined by the behavior near T_c of the *bare* ratio

$$\kappa_d^2 = \frac{m_\psi^2}{m_A^2}. \tag{9}$$

It must be observed that $m_\psi^2 \sim t$, where t is the reduced temperature. Since the following argument is valid to all orders, we are assuming that the critical temperature contains already all the fluctuations. Thus, in the bare mass we are using a *renormalized* critical temperature.[a] If we look at the scaling of the bare photon mass in the GL model, we see that we have also $m_A^2 \sim t$. This is the main motivation of what we will call Kleinert's scaling [8], where κ_d is constant, just like the Ginzburg constant κ in the GL model.

We define the renormalized fields as $\psi_r = Z_\psi^{-1/2}\psi$ and $\mathbf{h}_r = Z_h^{-1/2}\mathbf{h}$. From the Ward identities we conclude that the term $m_A^2\mathbf{h}^2/2$ does not renormalize. Thus, we obtain $m_{A,r}^2 = Z_h m_A^2$. The renormalization of the remaining parameters is given by $m_{\psi,r}^2 = Z_m^{-1}Z_\psi m_\psi^2$, $u_{\psi,r} = Z_{u_\psi}^{-1}Z_\psi^2 u$ and $e_{d,r}^2 = Z_h e_d^2$. We observe from the renormalization of e_d^2 that the charge e is not renormalized in the dual model. Let us introduce the dimensionless renormalized couplings $f_d = e_d^2/m_\psi$ and $g_\psi = u_{\psi,r}$. In order to obtain the flow equations we need to differentiate the renormalized quantities with respect to $m_{\psi,r}$, keeping fixed the bare quantities that *do not depend on* t. The following flow equations are

[a]For instance, at 1-loop the critical temperature would be corrected by a term proportional to the ultraviolet cutoff.

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easily obtained:

$$m_{\psi,r}\frac{\partial m_{A,r}^2}{\partial m_{\psi,r}} = (\eta_h + 2 + \eta_m - \eta_\psi) m_{A,r}^2, \tag{10}$$

$$m_{\psi,r}\frac{\partial f_d}{\partial m_{\psi,r}} = (\eta_h + 1 + \eta_m - \eta_\psi) f_d, \tag{11}$$

where the RG functions η_h, η_m and η_ψ are defined by

$$\eta_h = m_{\psi,r}\frac{\partial \ln Z_h}{\partial m_{\psi,r}}, \tag{12}$$

$$\eta_m = m_{\psi,r}\frac{\partial \ln Z_m}{\partial m_{\psi,r}}, \tag{13}$$

$$\eta_\psi = m_{\psi,r}\frac{\partial \ln Z_\psi}{\partial m_{\psi,r}}. \tag{14}$$

It is straightforward to see that the infrared stable fixed point corresponds to $f_d^* = 0$. Therefore, the correlation length exponent is simply given by $\nu \approx 0.67$. Near the infrared stable fixed point, Eq. (10) becomes

$$m_{\psi,r}\frac{\partial m_{A,r}^2}{\partial m_{\psi,r}} \approx \frac{1}{\nu} m_{A,r}^2. \tag{15}$$

This means that $m_{A,r}^2 \sim m_{\psi,r}^{1/\nu}$. Since $m_{A,r} = \lambda^{-1}$ we have the *exact* penetration depth exponent $\nu' = 1/2$. Thus, in Kleinert's scaling the exponent ν' remains classical to *all orders*.

Other scalings corresponding to different physical situations are also possible in the dual model. For example, the scaling that gives the XY universality class corresponds to taking m_A *fixed*, that is, with no dependence on t. In this scaling $\kappa_{d,r} \to 0$ as $t \to 0$ and the penetration depth exponent is given by $\nu' = \nu/2$. This scaling is well known experimentally [1].

Experimentally, Kleinert's scaling seems to have been probed by the authors of Ref. [15] and more recently in Ref. [14]. Both experiments used YBCO thin films at *optimal doping*. At optimal doping, 3D fluctuations are still more relevant, even in YBCO thin films, due to the strong coupling between the CuO planes.

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4 Conclusion

Let us summarize the main lessons of this paper. First, the charged fixed point corresponds to a strong-coupling problem which is very difficult to be studied directly in the GL model where special perturbative or non-perturbative techniques must be employed. Duality gives in this case a powerful access towards the understanding of this problem since it provides a weak-coupling realization of the strong-coupling limit of the GL model. Second, the tricritical point of the superconductor is a Lifshitz point, and that is the reason why the anomalous dimension of the superconductor is negative.

An interesting perspective is the use of duality in a nonzero field problem. This approach is considerably more difficult since the phase diagram is richer. However, it can be hoped that the duality approach would also be helpful in this context.

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