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NON-EQUILIBRIUM WORLDLINE DUALITY IN CONDENSED MATTER

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The dual "worldline" description of adiabatic Ginzburg-Landau field theory near a phase transition in terms of quasi-Brownian strings and loops is well understood, particularly through the work of Hagen Kleinert and his coworkers. In reality, the implementation of a transition is intrinsically non-equilibrium. We sketch how time-dependent Ginzburg-Landau theory leads to a modified dual string picture. A causal bound on the growth of unstable string prevents the uncontrolled proliferation of string (the Shockley-Hagedorn transition) suggested by the adiabatic approximation.

1 Introduction

The dual worldline approach to time-independent Ginzburg-Landau (TIGL) theory is well understood, significantly through the work of Hagen Kleinert and his coworkers. From the early days of lattice models [1] it has been appreciated that time-independent Ginzburg-Landau (TIGL) theory has a dual representation in terms of Brownian strings. A detailed discussion is given in the textbook by Kleinert [2] and, more recently, in lecture notes by Schakel [3]. An intrinsic ingredient of all these calculations is that they are performed for systems in *equilibrium*, in which the temperature T is fixed at values closer and closer to the critical temperature T_c. For the case that interests us, that of continuous transitions, the instabilities that characterise the transitions are, in this adiabatic dual picture, a consequence of the uncontrolled proliferation of string.

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In practice, transitions occur in a finite, often short, time in which the temperature T crosses T_c from its initial value. The adiabatic approximation is only valid away from the transition, when the field can adjust to the changing environment. Close to the transition the adiabatic regime is replaced by an impulse regime, in which the field falls out of step with what its equilibrium behavior would be. In particular, the correlation length cannot grow faster than the relevant speed (of sound) at which the field can order itself and, as a result, is unable to diverge [4,5]. We would expect a realistic dual worldline description in terms of strings to show a similar causal bound that will provide a limit to their proliferation as we pass through a transition.

The question that we shall begin to address here is how the intrinsically non-equilibrium theory has a dual representation in terms of strings and loops, and how the transition is manifest.

2 Equilibrium Ginzburg-Landau Duality

We begin with a brief recapitulation of equilibrium dual theory. The simplest condensed matter TIGL theory is that of a single complex field ϕ, with the Ginzburg-Landau free energy

$$F(T) = \int d^3x \left(\frac{\hbar^2}{2m} |\nabla\phi|^2 + \alpha(T)|\phi|^2 + \beta|\phi|^4 \right), \tag{1}$$

in which the chemical potential $\alpha(T) = \alpha_0 \epsilon(T)$, where $\epsilon(T) = (T/T_c - 1)$, vanishes at T_c. We envisage changing $\alpha(T)$ through an external cooling of the system or a change in the pressure of the system that leads to a change in T_c. Such an energy provides a reasonable description of superfluid ^{4}He, a simplified model for ^{3}He and a good description of the scalar sector of low-T_c superconductors.

It is convenient to work in spatial units of $\xi_0 = \sqrt{\hbar^2/2m\alpha_0}$ when, on rescaling the field, the self-coupling and the temperature (but leaving ϵ unchanged),

$$\bar{F}(T) = \int d^3x \left(|\nabla\phi|^2 + \epsilon(T)|\phi|^2 + \bar{\beta}|\phi|^4 \right). \tag{2}$$

At temperature $T_0 > T_c$ the *free-field* correlation function that follows from

$$\bar{F}_0(T_0) = \int d^3x \left(|\nabla\phi|^2 + \epsilon(T_0)|\phi|^2 \right) \tag{3}$$

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is

$$\langle\phi(\mathbf{x})\phi^*(\mathbf{0})\rangle = G_0(r) = \int d\!\!{}^-{}^3k\, e^{i\mathbf{k}\cdot\mathbf{x}}P(k), \tag{4}$$

($r = |\mathbf{x}|$) in which the power spectrum

$$P(k) = \frac{1}{\mathbf{k}^2+\epsilon(T_0)} = \int_0^\infty d\tau\, e^{-\tau k^2}\, e^{-\tau\epsilon(T_0)} \tag{5}$$

has the usual representation in terms of the Schwinger proper-time. In turn, this gives

$$G_0(r) = \int_0^\infty d\tau \left(\frac{1}{4\pi\tau}\right)^{3/2} e^{-r^2/4\tau}\, e^{-\tau\epsilon(T_0)}. \tag{6}$$

$G_0(0)$, necessary for loops, diverges from the UV singularities at $\tau = 0$, but these can be regulated with a fixed cutoff. Typically, we choose a cutoff of order unity in units of ξ_0, which will be implicit throughout.

The dual picture is obtained by observing that $(1/4\pi\tau)^{3/2}e^{-r^2/4\tau}$ is the probability distribution for a Brownian "worldline" or, more usefully, a "polymer" path $\mathbf{x}(\tau)$ beginning at the origin $\mathbf{x}(0) = 0$ and ending at $\mathbf{x}(\tau) = \mathbf{x}$,

$$\left(\frac{1}{4\pi\tau}\right)^{3/2} e^{-r^2/4\tau} = \int_{\mathbf{x}(0)=\mathbf{0}}^{\mathbf{x}(\tau)=\mathbf{x}} \mathcal{D}\mathbf{x}\,\exp\left[-\int_0^\tau d\tau'\,\frac{1}{4}\left(\frac{d\mathbf{x}}{d\tau'}\right)^2\right]. \tag{7}$$

If we think of the path as having step length unity (in units of ξ_0) then τ is proportional to the length of the path [3] and we shall use τ and path length synonymously. Then $G_0(r)$ is the sum over string paths of all lengths,

$$G_0(r) = \int_0^\infty d\tau \int_{\mathbf{x}(0)=\mathbf{0}}^{\mathbf{x}(\tau)=\mathbf{x}} \mathcal{D}\mathbf{x}\, e^{-S_{\rm eq}[\mathbf{x},\tau,\epsilon(T_0)]}, \tag{8}$$

where $S_{\rm eq}[\mathbf{x};\tau,\epsilon(T_0)]$ is the equilibrium Euclidean action

$$S_{\rm eq}[\mathbf{x},\tau,\epsilon(T_0)] = \int_0^\tau d\tau' \left[\frac{1}{4}\left(\frac{d\mathbf{x}}{d\tau'}\right)^2 + \epsilon(T_0)\right]. \tag{9}$$

The free-field partition function

$$\mathcal{Z} = \int \mathcal{D}\phi\,\mathcal{D}\phi^*\; e^{-\bar{F}_0} = \exp[-\,\mathrm{tr}\,\ln(-\nabla^2+\epsilon(T_0))] \tag{10}$$

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is just that of a gas of free orientable polymer or string *loops*, whose lengths are labelled by τ, with the one-loop partition function

$$\ln \mathcal{Z} = \int_0^\infty \frac{d\tau}{\tau} \left(\frac{1}{4\pi\tau}\right)^{3/2} e^{-\tau\epsilon(T_0)}. \tag{11}$$

$\epsilon(T_0)$ is understood as the energy/unit length or tension of the string. The factor $\propto \tau^{-5/2}$ in (11) is the length distribution for Brownian strings of length τ. As we drive $T_0 \to T_c+$ and $\epsilon(T_0)$ to zero it is easier to create strings. If, formally, we take T_0 below T_c then the onset of *negative* tension makes long loops of this unstable string overwhelmingly favoured and the condensation of loops that follows is understood by condensed matter physicists as a Shockley-Feynman transition, and by quantum field theorists as a Hagedorn transition.

The average loop length is

$$\langle\tau\rangle = -\frac{d \ln \mathcal{Z}}{d\,\epsilon(T_0)} = \frac{\int_0^\infty d\tau\,(4\pi\tau)^{-3/2}\,e^{-\tau\epsilon(T_0)}}{\int_0^\infty d\tau\tau^{-1}\,(4\pi\tau)^{-3/2}\,e^{-\tau\epsilon(T_0)}}. \tag{12}$$

Yet again, forcing T below T_c makes $\langle\tau\rangle$ diverge from the IR divergence of the integrals at large τ. A formal cutoff at $\tau = \tau_{\max}$ leads to $\langle\tau\rangle = O(\tau_{\max})$ once this happens.

The incorporation of the $\bar{\beta}|\phi|^4$ field self-interaction into equilibrium dual string theory is implemented by the introduction of a repulsive steric string interaction at the points where paths or strings cross, to give an additional action of the form [2]

$$S_{\rm eq}^{\rm int}[\mathbf{x},\tau] = -\bar{\beta}\int_0^\tau d\tau_k d\tau_l\;\delta[\mathbf{x}(\tau_k) - \mathbf{x}(\tau_l)] \tag{13}$$

for strings parameterized by τ_k and τ_l. Although this changes the weight of string configurations, qualitatively the transition still occurs in this adiabatic picture because of the proliferation of strings.

3 Non-Equilibrium Ginzburg-Landau Duality

In practice, a change of phase is enforced by letting the temperature T change with time, or by varying the critical temperature T_c. Experiments with superconductors and (the neutron bombardment of) ^{3}He do the former, while

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pressure quenches of ^{4}He do the latter. If we write

$$\epsilon(t) = \epsilon(T(t)) = \frac{T(t)}{T_c(t)} - 1, \tag{14}$$

the adiabatic approximation for the "free" correlation function $G_0(r,t)$ at time t is

$$G_0^{\rm ad}(r,t) = \int_0^\infty d\tau \int_{\mathbf{x}(0)=\mathbf{0}}^{\mathbf{x}(\tau)=\mathbf{x}} \mathcal{D}\mathbf{x}\, e^{-S_{\rm eq}[\mathbf{x},\tau,\epsilon(t)]}, \tag{15}$$

in which we just make a straightforward substitution of the equilibrium $\epsilon(T_0)$ with $\epsilon(t)$ in $S_{\rm eq}[\mathbf{x},\tau,\epsilon(T_0)]$. This is treating the equilibrium pictures as a series of snapshots that can be run together as a continuous film. In this film string lengths increase uncontrollably as we cross the transition. In particular, $G_0^{\rm ad}(r,t)$ of (6) is simply calculable as the Yukawa correlator

$$G_0^{\rm ad}(r,t) = \frac{1}{4\pi r}\, e^{-r/\xi_{\rm ad}(t)}, \tag{16}$$

where, on rescaling, $\xi_{\rm ad}(t) = \xi_0/\sqrt{\epsilon(t)}$ diverges as $T(t) \to T_c$. As we noted earlier, this cannot be the case, since causality alone prevents the correlation length diverging in a finite time. In terms of the dual picture this implies that the production of an infinity of strings in a finite time is equally prohibited.

To see how, we adopt the time-dependent Landau-Ginzburg (TDLG) equation for $\bar{F}$,

$$\frac{1}{\Gamma}\frac{\partial\phi}{\partial t} = -\frac{\delta\bar{F}}{\delta\phi} + \eta, \tag{17}$$

where η is Gaussian thermal noise, satisfying

$$\langle\eta(\mathbf{x},t)\eta^*(\mathbf{y},t')\rangle = 2T(t)\Gamma\delta(\mathbf{x}-\mathbf{y})\delta(t-t'). \tag{18}$$

Let us continue to consider the free-field case, $\bar{F} = \bar{F}_0$. In (17) the natural unit of time is $\tau_0 = 1/\alpha_0\Gamma$ and, in units of τ_0 and ξ_0, Eq. (18) becomes

$$\dot{\phi}(\mathbf{x},t) = -[-\nabla^2 + \epsilon(t)]\phi(\mathbf{x},t) + \bar{\eta}(\mathbf{x},t), \tag{19}$$

where $\bar{\eta}$ is the renormalized noise. The equal-time correlation function is constructed from the solution

$$\phi(\mathbf{k},t) = \int_{-\infty}^t dt'\, e^{-\int_{t'}^t dt''\,(\mathbf{k}^2+\epsilon(t''))}\bar{\eta}(\mathbf{k},t') \tag{20}$$

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for the spatial Fourier transform of ϕ as

$$\langle\phi(\mathbf{x},t)\phi^*(\mathbf{0},t)\rangle = G_0(r,t) = \int d\!\!{}^-{}^3k\, e^{i\mathbf{k}\cdot\mathbf{x}}P(k,t). \tag{21}$$

$P(k,t)$ has a representation in terms of the Schwinger proper-time τ as

$$P(k,t) = \int_0^\infty d\tau\,\bar{T}(t-\tau/2)\,e^{-\tau k^2}\,e^{-\int_0^\tau d\tau'\ \epsilon(t-\tau'/2)}, \tag{22}$$

where $\bar{T}$ is the renormalized temperature. Assuming that, for sufficiently negative $t<0$ in the past, $T(t)=T_0$, then[a] $\bar{T}(t)=T(t)/T_0$. In turn, this gives

$$G_0(r,t) = \int_0^\infty d\tau\,\bar{T}(t-\tau/2)\left(\frac{1}{4\pi\tau}\right)^{3/2} e^{-r^2/4\tau}\,e^{-\int_0^\tau d\tau'\ \epsilon(t-\tau'/2)}. \tag{23}$$

However, $G_0(r,t)$ can still be expressed in terms of paths, as

$$G_0(r,t) = \int_0^\infty d\tau\,\bar{T}(t-\tau/2)\int_{\mathbf{x}(0)=\mathbf{0}}^{\mathbf{x}(\tau)=\mathbf{x}} \mathcal{D}\mathbf{x}\,e^{-S[\mathbf{x},\tau,t]}, \tag{24}$$

where $S[\mathbf{x},\tau,t]$ is not the equilibrium action $S_{\rm eq}[\mathbf{x},\tau,\epsilon(t)]$, but

$$S[\mathbf{x},\tau,t] = \int_0^\tau d\tau'\ \left[\frac{1}{4}\left(\frac{d\mathbf{x}}{d\tau'}\right)^2 + \epsilon(t-\tau'/2)\right]. \tag{25}$$

This differs significantly from (15), which would replace $\epsilon(t-\tau'/2)$ in (25) with $\epsilon(t)$. The integrated tension $\int_0^\tau d\tau'\epsilon(t-\tau'/2)$ does *not* vanish at the transition time. Since it is the uniform vanishing of tension which triggers the avalanche of string production we see already that it will not happen in this case. Also, because of $\bar{T}(t-\tau/2)$, the paths are no longer Brownian in their length distribution.

4 Examples

4.1 The Instantaneous Quench

As a simple, if unrealistic, demonstration that transitions implemented in a finite time do not display singular dual behavior we consider the case of an *instantaneous* quench at time $t=0$ from a temperature T_0 above the

[a] This slightly cumbersome formalism hides the fact that, conventionally, the fields are rescaled so that the correlation function contains an additional factor of temperature.

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critical temperature T_c to absolute zero. That is, $\epsilon(t) = \epsilon(T_0)\theta(-t)$. Simple calculation shows that $G_0(r,t)$ takes the form of (6) for $t < 0$, whereas for $t > 0$

$$G_0(r,t) = e^{2tT_0/T_c} \int_{2t}^{\infty} d\tau \int_{\mathbf{x}(0)=\mathbf{0}}^{\mathbf{x}(\tau)=\mathbf{x}} \mathcal{D}\mathbf{x}\, e^{-S_{\mathrm{eq}}[\mathbf{x},\tau,\epsilon(T_0)]} \tag{26}$$
$$= e^{2tT_0/T_c} \int_{2t}^{\infty} d\tau \left(\frac{1}{4\pi\tau}\right)^{3/2} e^{-r^2/4\tau}\, e^{-\tau\epsilon(T_0)}.$$

After the transition we have a representation in terms of positive tension paths at the *initial* temperature T_0. The unstable (negative tension) paths, for which $\tau < 2t$, are *totally excluded*. Instead, the instabilities are encoded in the non-singular exponential prefactor.

This lack of IR singular behavior is made even more explicit in a saddle-point approximation for $G_0(r,t)$. For $r^2 > 4\epsilon(T_0)t$ we find

$$G_0(r,t) \sim e^{2tT_0/T_c} \frac{1}{4\pi r}\, e^{-r/\xi}, \tag{27}$$

where $\xi = \xi(T_0)$ remains frozen in at its initial equilibrium value. Unlike $\xi_{\mathrm{ad}}(t)$ there is no divergence of ξ as we cross $T = T_c$ in this abrupt way.

For the equilibrium theory, $\langle\tau\rangle$ of (12) can also be expressed as

$$\langle\tau\rangle = \frac{G_0(0)}{2G_0''(0)}, \tag{28}$$

where the prime denotes differentiation with respect to r. If we adopt the same definition out of equilibrium, the benign effect of the quench to $T_f = 0$ is again apparent in that $\langle\tau\rangle_t$ agrees with (12) for $t < 0$, but for $t > 0$ we have

$$\langle\tau\rangle_t = \frac{\int_{2t}^{\infty} d\tau\, (4\pi\tau)^{-3/2}\, e^{-\tau\epsilon(T_0)}}{\int_{2t}^{\infty} d\tau\tau^{-1}\, (4\pi\tau)^{-3/2}\, e^{-\tau\epsilon(T_0)}}. \tag{29}$$

The exponential prefactors have cancelled to reproduce the equilibrium result (12), again at the initial temperature $T = T_0$, but for the absence of unstable loops with a length less than $2t$, to which there is now no reference. Because of the exponential damping of a long string, the dominant string length is $\tau = 2t$.

There is no question of an IR divergence of loop length as naively suggested by the equilibrium theory. We understand the stability of loops with length $\tau > 2t$ in (29) as a causal bound. In our units, negative tension can only

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propagate at speed $c = 1$, which happens to be the cold speed of sound in the ϕ field.

This is reinforced by an extension to non-zero final temperature T_f

$$\epsilon(t) = \begin{cases} \epsilon(T_0) > 0, & t < 0, \\ \epsilon(T_f) < 0, & t > 0. \end{cases} \tag{30}$$

For $t > 0$ we now find

$$\begin{aligned} G_0(r,t) = e^{2t(T_0 - T_f)/T_c} \int_{2t}^{\infty} d\tau \int_{\mathbf{x}(0)=\mathbf{0}}^{\mathbf{x}(\tau)=\mathbf{x}} \mathcal{D}\mathbf{x}\, e^{-S_{\rm eq}[\mathbf{x},\tau,\epsilon(T_0)]} \\ + \frac{T_f}{T_0} \int_0^{2t} d\tau \int_{\mathbf{x}(0)=\mathbf{0}}^{\mathbf{x}(\tau)=\mathbf{x}} \mathcal{D}\mathbf{x}\, e^{-S_{\rm eq}[\mathbf{x},\tau,\epsilon(T_f)]}. \end{aligned} \tag{31}$$

We see explicitly how the limited length $\tau < 2t$ of string with negative tension $\epsilon(T_f) < 0$ prevents such string from giving a divergent contribution.

Small unstable loops are no longer precluded, but their contribution is finite. The dominant length remains $\tau = 2t$.

4.2 Slower Quenches

Although an instantaneous quench is impossible, more general quenches show similar qualitative behavior.

Specifically, suppose that $\epsilon(t)$ decreases monotonically, with a single zero $\epsilon(0) = 0$ at time $t = 0$. Then strings of limited length $\tau < 2t$, for which $\epsilon(t - \tau/2) < 0$, have "negative" tension. However, for $\tau > 2t$, for which $\epsilon(t - \tau/2) > 0$, strings have segments with negative and positive tension. There is a causal bound $c = 1$ on the speed at which instability can propagate along a string. The string with negative tension, with a contribution that is independent of τ, gives a prefactor growing at least exponentially in time. The effect is to leave only stable string of length $\tau > 2t$ in the τ integral for $\tau > 2t$. This boundedness on unstable string prevents the unlimited production of string suggested by the adiabatic approximation.

Examples of linear quenches are given in our earlier work [6-8], but the framework of duality was not developed in them (although it was identified in the last of them). Rather than go into any detail, we can generalise further by attempting to accommodate the self-interaction which, in itself, leads to a modified mass-behavior $\epsilon(t)$.

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5 Back-Reaction

The exponential growth of $G_0(r,t)$ with time in (27) can only be accommodated for a very short period, since $\langle|\phi|^2\rangle = G_0(0,t)$ must be constrained by the value of the order parameter $\langle|\phi|\rangle$ after the transition, equal to $\sqrt{\bar{\beta}^{-1}/2}$ in the absence of corrections. A rough guide to the maximum time $\bar{t}$, for which the free-field approximation is valid, is that $G_0(0,\bar{t}) = \bar{\beta}^{-1}/2$. As t approaches $\bar{t}$, and thereafter, the reaction of the field with itself will cut off the exponential growth.

It is difficult to see how the dual string picture will survive at later times without further approximation. One indication is through a mean-field (or large-N) approach in which the linear nature of the theory is maintained self-consistently [9]. In this approach (19) is replaced by

$$\dot{\phi}(\mathbf{x},t) = -[-\nabla^2 + \epsilon_{\rm eff}(t)]\phi(\mathbf{x},t) + \bar{\eta}(\mathbf{x},t), \tag{32}$$

where

$$\epsilon_{\rm eff}(t) = \epsilon(t) + p\bar{\beta}\langle|\phi|^2\rangle = \epsilon(t) + p\bar{\beta}G(0,t), \tag{33}$$

and $G(0,t)$ is determined self-consistently[b] from (32).

The assumed single zero of $\epsilon(t)$ at $t=0$ will lead to a zero of $\epsilon_{\rm eff}(t)$ at $t \approx 0$, and the previous analysis applies. With only a finite length of string with negative tension, there is no singular behavior as we cross the transition.

To have a quantitative estimate of the effect of back-reaction we make a Gaussian approximation for $G(0,t)$ by expanding about the zero of $\epsilon_{\rm eff}(t)$. Assuming that the quench is not too rapid, we find that, for $t \le \bar{t}$, the average loop length is

$$\langle\tau\rangle_t \approx \frac{\int_0^\infty d\tau\, \bar{T}(t-\tau/2)(4\pi\tau)^{-3/2}\, e^{-(\tau-2t)^2|\epsilon'(0)|/4}}{\int_0^\infty d\tau\tau^{-1}\, \bar{T}(t-\tau/2)(4\pi\tau)^{-3/2}\, e^{-(\tau-2t)^2|\epsilon'(0)|/4}}. \tag{34}$$

In this approximation the prefactors encoding the unstable strings cancel, and we need no information about the self-consistent mass, but for the fact that $\epsilon_{\rm eff}(t) \to 0$ at large times to stop $G(0,t)$ from growing. We see the peak at $\tau = 2t$ in the length distribution moving clear of the UV endpoint behavior. This continues for later times and $\langle\tau\rangle_t = O(t)$ once the temperature

[b]The coefficient p depends on whether we adopt a mean-field (Hartree) approximation or a large-N limit for $N=2$.

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has become low enough that $\bar{T}(t)$ has suppressed the UV singular behavior. Details are given in our earlier work [7,8].

We stress that this discussion has been restricted to the "first quantised" dual representation of the Ginzburg-Landau theory. However, it is a familiar result from a different viewpoint. For a linear system like (32) it can be shown [10,11] that the mean loop length is

$$\langle\tau\rangle_t = \frac{1}{4\pi n(t)}, \tag{35}$$

where $n(t)$ is the density of *line zeroes* of the complex field ϕ. The relevance of this is that, at later times, the global *vortices* of this $U(1)$ scalar theory can be identified by the line zeroes of their cores [12]. The linear growth of *dual* loop lengths with time corresponds to a $t^{1/2}$ behavior for *ϕ-field line-zero* separation and, when vortices become well-defined, vortex separation. This scaling behavior can be justified [13] for the decay of vortices of ^{4}He, and is used to determine initial vortex densities at the ^{4}He transition [14].

6 Conclusions

Our conclusions are simple. In the adiabatic approximation for the (world-line) string representation of scalar field theory, the transition is signaled by the proliferation of string at a Shockley-Feynman/Hagedorn transition, as it becomes unstable (negative tension). In reality, transitions implemented in a finite time are intrinsically non-adiabatic and this picture breaks down. In a more realistic dual string representation at time t after the transition begins, there is a causal constraint at the rate at which instability can propagate along strings. With only short strings and short segments of strings unstable, there is no Shockley-Feynman/Hagedorn transition, since this relies on the instability of arbitrarily long strings.

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