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METRIC AND CONNECTION: KINEMATIC AND DYNAMIC SOLUTIONS OF THE SPACE PROBLEM

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We compare metric theories to bi-metric theories, to theories with teleparallelism, and to metric-affine (Einstein-Cartan) theories of gravitation in order to discuss the implications arising from the equivalence principle for the related space and energy problems.

1 Introduction

All classical local geometric theories of gravitation are based on the assumption that the space-time manifold is primarily or can secondarily be endowed with a Lorentz metric, i.e. with a symmetric tensor field of signature $(+,-,-,-)$. This is even true for purely affine theories such as the Einstein-Schrödinger theory [1,2], where the geometry is specified by a connection as basic variable, while the metric is only of secondary importance. In the final analysis, a metric structure is always needed in order to arrive at a theory which is physically interpretable. Only such a structure allows us to introduce the notions of spatial distance, time interval, angle, and relativistic velocity.

As a consequence of the principle of equivalence, however, the metric has to be related to the gravitational field. In contrast to the special-relativistic approach, it cannot be introduced *a priori*, but has to be specified in accordance with the other unrenouncable geometric structure, the connection. In

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other words, a theory satisfying the principle of equivalence has to be established in such a way that it solves the so-called space problem (see Refs. [3,4]). First of all, this means to find a gravitational dynamics that does not clash with the "kinematic" conditions ensuring the compatibility between metric and connection.

As far as the kinematic part of the Weyl-Cartan space problem is concerned, it was shown by Schrödinger [2], that the relationship

$$\nabla_l g_{ik} = 0 \tag{1}$$

between the metric g_{ik} and the connection Γ^i_{kl} defined by ∇_l is a sufficient condition for their compatibility.[a]

In view of the dynamic aspect of the space problem, there exists a further argument in favor of Eq. (1). It says that the coupling of spinorial matter to geometric structures which are related to gravitation requires to assume the validity of Eq. (1) [6].[b] Indeed, it follows from (1) that

$$\Gamma^i_{kl} = \left\{ {i \atop kl} \right\} + \Gamma^i_{[kl]} + g^{im} g_{kr} \Gamma^r_{[lm]} + g^{im} g_{lr} \Gamma^r_{[km]} = \left\{ {i \atop kl} \right\} + K^i_{kl}, \tag{2}$$

where $\left\{ {i \atop kl} \right\}$ is the Christoffel connection and K^i_{kl} the contorsion which is antisymmetric in the first two indices, $K_{ikl} = -K_{kil}$. In the anholonomic version, Eq. (2) reads

$$\Lambda^A{}_{Bl} = \gamma^A{}_{Bl} + K^A{}_{Bl}\,, \tag{3}$$

$$\gamma_{ikl} = -\gamma_{kil}, K_{ikl} = -K_{kil}\,, \tag{4}$$

where $\gamma^A{}_{Bl}$ are the Ricci rotation coefficients, $\gamma^A{}_{Bl} = h^A{}_k h_{B||l}{}^k$, and the last expression in Eq. (3) is defined as $K^A{}_{Bl} = h^A{}_i h_B{}^k K^i_{kl}$; both are antisymmetric in the first two indices. As a consequence, the anholonomic components of the internal connection are antisymmetric in the first two indices, too:

$$\Lambda_{ikl} = -\Lambda_{kil}\,. \tag{5}$$

Exploiting the one-to-two correspondence of the Lorentz group $O(3,1)$ to the unimodular group $SL(2,C)$, this recovers the usual spinor formalism, where

[a] For physical reasons, Einstein [5] considered Eq. (1) even as a necessary condition.

[b] By means of another line of arguments, this was also shown in Refs. [7,8]. Even more, it was demonstrated there how the usual spinor formalism can be generalized for the case that the nonmetricity is of the Weyl form. We shall confine ourselves to spaces with a vanishing nonmetricity. But all our arguments given in this paper can be generalized to spaces with a nonvanishing Weyl nonmetricity, i.e. to semi-metric spaces.

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$\gamma_{\alpha\beta}$ and $\gamma_{\dot\alpha\dot\beta}$ are the "metrics" in the spin spaces:

$$\gamma_{\alpha\beta} = -\gamma_{\beta\alpha}, \gamma_{\dot\alpha\dot\beta} = -\gamma_{\dot\beta\dot\alpha}, \gamma_{\alpha\beta,l} = \gamma_{\dot\alpha\dot\beta,l} = 0, \Lambda_{\alpha\beta l} = \Lambda_{\beta\alpha l}, \Lambda_{\dot\alpha\dot\beta l} = \Lambda_{\dot\beta\dot\alpha l} \,. \quad (6)$$

These conditions, here justified with regard to a possible gravitational dynamics, must of course be recovered or, at least, not be violated by the full gravitational dynamics.

2 Metric Theories

In metric theories the space problem is solved *a priori* by assuming a Riemannian space-time, i.e. by assuming a four-dimensional manifold with a Lorentz metric, where the connection given by the metric and their first derivatives satisfies (1) identically. In this case, the field equations for the metric cannot conflict with (1). Then the free gravitational Lagrange density is formed from g_{ik} and $\partial_l g_{ik}$

$$\mathbf{L}_f = \mathbf{L}_f(g_{ik}, \partial_l g_{ik}, \partial_l \partial_m g_{ik}), \quad (7)$$

while the matter Lagrange density depends on both the gravitational and matter variables, g_{ik} and $\phi^{(A)}$; imposing, as an implication of the principle of equivalence, the principle of minimal coupling, one generally has

$$\mathbf{L}_m = \mathbf{L}_m(g_{ik}, \partial_l g_{ik}, \phi^{(A)}, \partial_l \phi^{(A)}) \,. \quad (8)$$

The Euler variation of the action integral

$$I = \int (\mathbf{L}_f + 2\kappa \mathbf{L}_m) d^4x \,, \quad (9)$$

by the matter field provides the matter field equations

$$\frac{\delta \mathbf{L}_m}{\delta \phi^{(A)}} = 0 \quad (10)$$

and by the metric the gravitational equations

$$G_{ik} = -\kappa P_{ik} \,, \quad (11)$$

where the Einstein tensor is defined by

$$G_{ik} = \frac{\delta \mathbf{L}_f}{\delta g^{ik}} \quad (12)$$

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and the metric energy-momentum tensor of matter reads

$$P_{ik} = \frac{2}{\sqrt{-g}} \frac{\delta \mathbf{L}_m}{\delta g^{ik}} \,. \tag{13}$$

Furthermore, assuming that all field equations are satisfied, one obtains [9],

$$\delta^* \mathbf{L} + (\mathbf{L}\xi^m)_{,m} = 0 \tag{14}$$

with $\delta x^i := \xi^i$, the Lie differential $\delta^* g^{ik}$ of g^{ik},

$$\delta^* g^{ik} = g^{sk} \xi^i{}_{,s} + g^{is} \xi^k{}_{,s} \tag{15}$$

and the corresponding Lie differential $\delta^* \phi^{(A)}$ of $\phi^{(A)}$. For $\xi^i = \alpha^i = \text{const.}$ Eq. (14) yields the Noether identity

$$\left(P_i{}^k + \frac{1}{2\kappa} t_i{}^k \right)_{,k} \cong \nabla_k P_i{}^k = 0 \,, \tag{16}$$

where $t_i{}^k$ is the gravitational energy-momentum pseudo-tensor. This equation is equal to the contracted Bianchi identities when calculated via the field equations (16).

Following Lorentz [10], one can read the field equations (11) as the statement that the total metric energy-momentum density of matter and gravity is equal to zero. That such a reading of (11) has a physical meaning was shown for Einstein's GRT, where $\mathbf{L}_f = \sqrt{-g}R$, and for fourth-order gravitational equations, where $\mathbf{L}_f = \sqrt{-g}(R + \alpha R_{ik} R^{ik} + \beta R^2)$ [11,12]. For the recent development of the topic see Ref. [13] and the papers quoted therein.

Purely metric theories are insofar physically completely satisfying as they guarantee the following three points: (i) they solve the space problem; (ii) they are in agreement with the principles of equivalence and general relativity; (iii) their "conservation" laws given by Noether's theorem follow, via the field equations, automatically from the Bianchi identities. Otherwise, and that is their disadvantage, as a matter of these principles, they do not allow for the existence of a genuine law of energy-momentum conservation.

3 Bi-Metric Theories

Bi-metric theories have in addition to the Riemann-Einstein metric g_{ik}, a second metric $\overline{g}_{ik}$. Such theories go back to ideas of Rosen [14], Band [15], and Papapetrou [16], but their satisfying elaboration was only given by

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Kohler [17,18].[c] In this theory, the Weyl-Cartan space problem is solved in the same manner as in the uni-metric theories discussed above (both metrics g_{ik} and $\overline{g}_{ik}$ are presupposed to satisfy Eq. (1)). And, even more, since the second metric $\overline{g}_{ik}$ is assumed to be a pseudo-Euclidean one, one has 10 Killing vectors such that also the Helmholtz-Lie space problem is solved.

Corresponding to the two metrics, there exist the two (Riemann-Christoffel) connections Γ^i_{kl} and $\overline{\Gamma}^i_{kl}$, where the latter vanishes in pseudo-Cartesian coordinates. Their difference is a tensor ρ^i_{kl} describing the gravitational field such that the connection Γ^i_{kl} is the sum of the inertial field $\overline{\Gamma}^i_{kl}$ and the gravitational field ρ^i_{kl}:

$$\Gamma^i{}_{kl} = \overline{\Gamma}^i{}_{kl} + \rho^i{}_{kl} \, . \tag{17}$$

Kohler specified the free Lagrangian as

$$\mathbf{L}_\kappa = \mathbf{L}_f(g_{ik}, \partial_l g_{ik}, \overline{g}_{ik}, \partial_l \overline{g}_{ik}) + \mathbf{L}_m(g_{ik}, \partial_l g_{ik}, \phi^{(A)}, \partial_l \phi^{(A)}) \, , \tag{18}$$

such that, as in the above uni-metric theories, the metric-energy-momentum tensor fulfills again the dynamical equation with respect to Γ^i_{kl}:

$$\nabla_k P_i{}^k = 0 \, . \tag{19}$$

But now, due to the existence of the second metric, this can be rewritten as

$$\overline{\nabla}_k \left(\sqrt{\frac{g}{\overline{g}}} (P_i{}^k + t_i{}^k) \right) = 0 \, , \tag{20}$$

where $\overline{\nabla}$ is the covariant derivative with respect to $\overline{\Gamma}^i_{kl}$, g is the determinant of g_{ik}, and $\overline{g}$ the determinant of $\overline{g}_{ik}$. In contrast to $t_i{}^k$ in Eq. (16), the expression $t_i{}^k$ arising in Eq. (20) is a tensor under general coordinate transformations.

The variation of Eq. (18) by g_{ik} provides 10 field equations, while its variation by $\overline{g}_{ik}$ leads to four additional equations. The latter ones can be interpreted as coordinate conditions soldering the two metrics; they are equivalent to Eq. (19). Thus, in this theory the dynamical Eq. (19), representing a conservation law according to Eq. (20), is no implication of the Bianchi identities and the field equations. This is the price one has to pay for the formulation of differential laws of conservation [19]. Except for this point, one finds the same situation as in the uni-metric theories considered above.

[c] See also Refs. [19,20].

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4 Theories with Teleparallelism (Einstein-Mayer-Type Theories)

Theories with teleparallelism consider the tetrad field $h^A{}_i(x^l)$ given by the coframe field $h^A = h^A{}_i dx^i$ as basic quantity; the metric is a secondary concept defined as

$$g_{ik} = \eta_{AB} h^A{}_i h^B{}_k \,, \tag{21}$$

with $\eta = \mathrm{diag}(1, -1, -1, -1)$. Since the $h^A{}_i$ are assumed to be fixed by the gravitational field equations, they define a teleparallelism in the Riemannian space. Thus the Weyl-Cartan space problem is solved in the same way as in GRT.

Let us start now from a Weitzenboeck Lagrange density which is a scalar density with respect to coordinate transformations but not invariant under local Lorentz transformations rule. One finds here a restricted invariance, namely an invariance with respect to global Lorentz transformations rule. To discuss the points in this paper under consideration we shall confine ourselves to the Einstein-Mayer class of Lagrangians [21-26]

$$\mathbf{L}_{EM} = \sqrt{-g}R + ahF_{Aik}F^{Aik} + bh\phi_A\phi^A \,, \tag{22}$$

where $h = \det(h^A{}_i) = \sqrt{-g}$, $R = g^{ik}R_{ik}$ is the Ricci scalar, a and b are numerical constants, the ϕ_A are defined as $\phi_A = h^i{}_A \gamma^m{}_{im}$, and the F_{Aik} are Cartan's anholonomy objects

$$F_{Aik} = h_{Ai,k} - h_{Ak,i} = h_A{}^l(\gamma_{lik} - \gamma_{lki}) \,, \tag{23}$$

with the Ricci rotation coefficients $\gamma_{ikl} = h^A{}_i h_{Ak;l}$. The Lagrangian (22) lies on the basis of Møller's tetrad theory of gravitation [27].[d] (From the viewpoint of a unified gravito-electromagnetic theory in the sense of Einstein's program of 1929, it is also discussed in Ref. [29]; however, in contrast to Einstein's approach, there, like in Møller's theory, an additional matter Lagrangian is introduced.) For historical reasons, we shall call the Lagrangian (22) "Einstein-Mayer Lagrangian", while the tetrad equivalent of the Einstein-Hilbert Lagrangian will be called "Møller Lagrangian". (For other metric-teleparallel theories, see, e.g. Kopczyński [30], Hayashi and Shirafuji [31]).

Now, in the anholonomic (Einstein-Cartan) representation of the space-time structure the reference systems, i.e. the "tetrads" $h^A{}_i(x^l)$, are the grav-

[d] In order to remove singularities from the theory of gravitation, later Møller [28] also introduced Lagrangians quadratic in the Weitzenboeck invariants.

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itational field variables. From the standpoint of the theories here under consideration, Einstein's GRT is a degenerate case, now following from (22) for $a = b = 0$.

In Refs. [26,27] one assumes for matter a Lagrange density depending only on the tetrads via the algebraic combination (21). This leads to the field equations

$$\frac{1}{h}\frac{\delta \mathbf{L}_{EM}}{\delta h^{Ai}} h^{A}{}_{k} = -\kappa T_{ik}\,, \tag{24}$$

with

$$\frac{\delta \mathbf{L}_{EM}}{\delta h^{Ai}} h^{A}{}_{k} - \frac{\delta \mathbf{L}_{EM}}{\delta h^{Ak}} h^{A}{}_{i} = 0\,. \tag{25}$$

The 10+6 equations (24) and (25) determine the 16 components of $h^{A}{}_{i}$ up to constant Lorentz rotations of the tetrads. Together with the Noether identities

$$H^{i}_{k,i} = F^{I}_{k;i} - F^{il}\gamma_{ilk}\,, \tag{26}$$

where $H_{ik} \equiv E_{ik} + \Theta_{(ik)}$ and $F_{ik} = \Theta_{[ik]}$, and together with Eq. (25) this leads again to the dynamical equations (16).

As argued in Refs. [32,33], in a Riemannian theory with Einstein-Cartan teleparallelism one finds quite a satisfying situation for the gravitational energy. One has a Lagrangian of canonical structure that is coordinate-covariant but, like Møller's Lagrangian, not covariant with respect to local Lorentz transformations rule. Thus the Lagrangian leads to field equations which do not satisfy the general principle of relativity. These are equations fixing the 16 components of the tetrads $h^{A}{}_{i}$ instead of their 10 combinations $g_{ik} = h^{A}{}_{i} h_{Ak}$, and the Hamiltonian $\mathbf{H}$ is given by the 00-component of the energy-momentum complex (which is a tensor with respect to the group of global Lorentz transformations rule lying on the basis of this theory).[e]

Similarly to the Kohler case, the dynamical equations of the theory with teleparalellism are not simply an implication of the Bianchi identities and the field equations, now given by Eq. (24). They result from the additional conditions (25) which specify the reference tetrads instead of the coordinates. Again, this is the price that one has to pay for having differential laws of

[e]In Refs. [32,33] such theories are considered as models realizing the local version of the Mach principle, according to which the cosmic matter distribution fixes the reference system.

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conservation. Or better, this is a part of the price, for, the other part is that one has even to sacrifice the local Lorentz covariance and, thus, the general principle of relativity.

5 Einstein-Cartan Theories

The more general case is considered in Ref. [34], where the matter Lagrange density depends independently on the 16 tetrad and the 64 connection components. Such a theory with teleparallelism can be considered as a constrained Einstein-Cartan theory which generally is formulated in spaces with nonvanishing curvature and torsion. Therefore, Einstein-Cartan theories can be regarded as a straightforward generalization of the Einstein-Mayer-type theories. Thus, let us turn to Einstein-Cartan theories.

As to the general principle of relativity, such theories resemble the purely metric theories: they are covariant under local Lorentz rule and general coordinate transformations. A great similarity is also found as to the energy-momentum problem. Indeed, starting from a Lagrange density of the form

$$\mathbf{L}_{EC} = \mathbf{L}_f(h^A{}_i, \Lambda^{AB}{}_i) + \kappa \mathbf{L}_m(h^A{}_i, \Lambda^{AB}{}_i, \Phi^{(A)}, \partial_l \Phi^{(A)}) \,, \tag{27}$$

where the tetrad and the connection fields, h and Γ, are regarded as independent variables, the Euler variation of $\Phi^{(A)}$, $h^A{}_i$, and $\Gamma^{AB}{}_i$ yields the matter field equations and the following gravitational equations (h denotes the determinant of $h^A{}_i$),

$$-\frac{\delta \mathbf{L}_f}{\delta h^A{}_i} = \kappa h T_A{}^i := \kappa \frac{\delta \mathbf{L}_m}{\delta h^A{}_i} \tag{28}$$

and

$$-\frac{\delta \mathbf{L}_f}{\delta \Lambda^{AB}{}_i} = \kappa h \tau_{AB}{}^i := \kappa \frac{\delta \mathbf{L}_m}{\delta \Lambda^{AB}{}_i} \,. \tag{29}$$

The source term in Eq. (28), $T_A{}^i$, is the canonical energy-momentum tensor, and the source term in Eq. (29), $\tau_{AB}{}^i$, represents the canonical spin-angular momentum tensor of the matter field. Therefore, as in GRT and its unimetric generalizations, Eq. (28) can be read as the statement that the total energy-momentum density is equal to zero. (However, this statement does not concern the total metric, but the canonical tensor.)

As to the fact that one has the same symmetries as in GRT, there does not exist a differential law of conservation. There exist dynamical equations

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which are a generalization of Eq. (19), and, for the class of theories under consideration in Refs. [35-38], it turns out that, provided the field equations are fulfilled, the generalized dynamical equations are again an automatic by-product of the Bianchi identities which hold in Riemann-Cartan space [39].

For spinless matter these generalized dynamical equations reduce to Eq. (19). This implies that spinless point particles move along geodesics of the Riemann-Christoffel connection, but not along autoparallels. Hagen Kleinert [40,41] considers this fact as an objection to the proposed class of Einstein-Cartan theories.

After mentioning a series of similarities between GRT and Einstein-Cartan theories, it should finally be stressed that there is a great difference as to the Weyl-Cartan space problem. The point is that in Einstein-Cartan theories the validity of Eq. (1) cannot kinematically be guaranteed. To solve the space problem now means to solve it by using an appropriate dynamic starting point. That is, one has to look for a Lagrangian such that the solutions of the corresponding gravitational field equations (28) and (29) satisfy Eq. (1), i.e. they yield solutions satisfying the constraints $\Lambda_{ABl} = -\Lambda_{BAl}$ (or $\Lambda_{\alpha\beta l} = \Lambda_{\beta\alpha l}$). From this point of view, for example, the Einstein-Hilbert Lagrangian $\sqrt{-g}R$, viewed as a functional of the metric and an arbitrarily generalized connection, has to be excluded from the consideration [42]; it leads to

$$\nabla_l g_{ik} = -2g_{is}\Gamma^s_{[kl]} + \frac{2}{3}(\Gamma_i g_{kl} + \Gamma_k g_{il}) \tag{30}$$

with the torsion vector $\Gamma_i = \Gamma^s_{is}$.

6 Conclusion

Physically interpretable theories need a metric. However, due to the principle of equivalence, the metric cannot be presupposed a priori. Taking this fact into consideration and rejecting any restriction of the principle of general relativity, one is necessarily led to purely metric theories like GRT or to Einstein-Cartan theories. Both suffer from the fact that the only energetic statement one can make consists in that the (metric and canonical, respectively) total energy-momentum density vanishes. As to the space problem, metric theories seem to be less problematic since for them it can be solved kinematically, while for Einstein-Cartan theories it has to be solved dynamically.

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