

Preface

The subject of this volume embodies a wide variety of research topics whose theoretical methods are based on path integrals, or more generally *Fluctuating Paths and Fields* – a heading which reflects best the numerous research areas of Hagen Kleinert to whom this *Festschrift* is devoted. The selection of authors and articles is guided by the difficult task of presenting a cross section of his research activities. With a few prominent exceptions, the contributors to this volume are his former and present students, postdocs, co-workers, or colleagues who have enjoyed working with him. With their articles they wish to celebrate his 60th birthday on June 15, 2001. We are especially grateful to the Physics Nobel Laureate Gerard 't Hooft for delivering the birthday colloquium at the Freie Universität Berlin.^a

The first part of the book contains recent advances in the path-integral description of quantum physics, covering and supplementing a subset of the topics discussed in Hagen Kleinert's widely-known textbook on *Path Integrals in Quantum Mechanics, Statistics, and Polymer Physics*. Among Kleinert's many contributions to this field, the best known is his path-integral solution of the three-dimensional hydrogen atom based on non-holonomic space-time transformations in 1979. The clue to the successful treatment was his deep knowledge of the relation between three-dimensional Coulomb and four-dimensional oscillator systems which was established in the context of celestial mechanics by Kustaanheimo and Stiefel. In standard quantum mechanics, this relation is rooted in the existence of the dynamical group $O(4, 2)$ of the hydrogen atom which Kleinert discovered in collaboration with the late Asim O. Barut during the years 1965 to 1967, when working on his Ph.D. thesis at the University of Colorado at Boulder. The main ideas for the path-integral solution are contained in a paper of 1979, written with his postdoc Ismail H. Duru. A fully consistent mathematical formulation, however, turned

^aThe abstract of this lecture is printed on p. xiii.

out to be quite subtle. In particular, it required an understanding of the path integration measure in curved space-time with torsion. The complete solution of the problem was found by Kleinert in the late eighties. Generalizing the theory allowed him to calculate path integrals of all systems for which the Schrödinger equation can be solved analytically. A full account of the theory can be found in his textbook on path integrals.

The impetus to tackle this fundamental problem of atomic physics goes back to Kleinert's discussions with Richard P. Feynman during a sabbatical at Caltech in 1973/74. As described in the Feynman biography by John and Mary Gribbin, "... Feynman had stopped teaching path integrals at a less advanced level, because he never derived a complete path integral description of the hydrogen atom, and was embarrassed by this failure. ... Kleinert ... not only solved the problem (much to Feynman's delight), but wrote a major textbook on the path integral approach, re-establishing path integrals as a research tool, not only conceptually useful but now capable of solving problems ..."^b Kleinert was invited to Pasadena by Murray Gell-Mann whom he had met before from 1968 to 1972 during various extended visits to CERN in Geneva, which was and continues to be one of the main centers for meeting important people (... such as his wife Annemarie). At that time, Gell-Mann was interested in a relation between current and constituent quarks derived by Kleinert and his colleagues at CERN, two of whom, Franco Buccella and Carlos A. Savoy, have written articles for this *Festschrift*. In Pasadena, Kleinert shared an office with Yuval Ne'eman whose pioneering work on $SU(3)$ had paved the way to Gell-Mann's development of a quark field theory and his successful prediction of the Ω^- particle. During that time Kleinert found a theoretical explanation for the algebra of Regge residues proposed by Ne'eman in collaboration with Cabibbo and Horwitz. This led to a deep friendship with Ne'eman, and even in his period as a Minister of Science and Technology in Israel, he found time to visit Kleinert in Berlin for discussing physics (see his contribution in this *Festschrift*).

The articles in Part I cover topics ranging from rigorous definitions of functional integrals through applications to group spaces, quasi-classical approximations, and semi-classical dynamics to numerical Monte Carlo evaluations of path integrals, which Kleinert, while working mainly analytically, often enjoyed performing on his personal computer at home. One of the edi-

^bJ. Gribbin and M. Gribbin, *Richard Feynman: A Life in Science* (Viking Penguin Books, London, 1997), pp. 215-216.

tors (W.J.) took part in a race of two good-old Atari computers against each other – one at the institute and the other at Kleinert’s home, the latter PC, of course, turning out to be newer and thus faster . . .

Part II of this volume collects articles on quantum field theory which has always been the backbone of Hagen Kleinert’s research. In fact, this was one of the reasons why Werner Theis, then one of the senior professors of theoretical physics at the Freie Universität Berlin, asked the 27 year-old Kleinert to join the faculty as an Associate Professor. After turning down two other offers for a full professorship elsewhere, he became a Full Professor in Berlin in 1976. Since then Kleinert, who grew up in Hannover and received his undergraduate education at the Technische Universität, became a real “Berliner”, refusing two further attractive offers from other universities in Germany and abroad. Kleinert’s early applications of field theoretic methods include the introduction of composite fields via path-integral transformations in models of elementary particle physics which he called “hadronization of quark theories” and, by analogy, in models of superconductors and superfluid Helium. Field theoretic methods involving gauge fields play a crucial role in Kleinert’s comprehensive work on defect-driven phase transitions which is collected in his two extensive monographs on *Gauge Fields in Condensed Matter* – Vol. I: *Superflow and Vortex Lines*, Vol. II: *Stresses and Defects*. In this context non-holonomic mappings are again of central importance. The articles in Part II discuss gauge theories, in particular quantum electrodynamics and quantum chromodynamics, and in some more detail ϕ^4 -field theories, where Kleinert and his collaborators contributed most over the recent years.

Variational perturbation theory and the resulting highly efficient scheme for resumming divergent perturbation series are discussed in Part III. This technique has its roots in a variational treatment of quantum-statistical partition functions in the path-integral representation developed by Kleinert in joint work with Feynman during his later sabbaticals at the Universities of California at Berkeley in 1979/80 and at Santa Barbara in 1982/83. After a considerable delay due to Feynman’s illness, the manuscript was finally finished and submitted for publication in *Physical Review* during Kleinert’s sabbatical stay at San Diego in 1985/86. This last period of intense scientific interactions with Feynman is also witnessed by the famous photographs of Feynman’s last office blackboards.^c The joint theory is an improvement of a 20 year-old less powerful approach found by Feynman himself. It is an amus-

^c*Feynman’s Office: The Last Blackboards*, in *Physics Today* **42**, 88 (February 1989).

ing fact of scientific history that, after such a long time, the same idea for an improved version was worked out independently also by Riccardo Giachetti and Valerio Tognetti in Florence in 1985/86. While subsequently the Italian group was mainly interested in applications to solid state physics, Kleinert pushed the method to higher orders and made it applicable to quantum field theory. Finally he arrived at an extremely powerful formulation of variational perturbation theory which may be viewed as a highly efficient resummation procedure for divergent, asymptotic perturbation series. Besides impressive applications in quantum mechanics and atomic physics, this has recently led to a successful treatment of ϕ^4 -field theories in the strong-coupling limit. The results are collected in a monograph on *Critical Properties of ϕ^4 -Theories*, written in collaboration with Verena Schulte-Frohlinde, who also contributes here.

The articles in Part IV deal with phase transitions and critical phenomena, many of them centering around explicit calculations of critical exponents of ϕ^4 -theories with and without anisotropies or quenched, random disorder. They have important applications to phase transitions in magnets and the λ -transition in liquid ^4He . Others are devoted to the related Ginzburg-Landau model and phase transitions in superconductors and superfluid ^3He where topological defects play a crucial role. Here, Kleinert's development of a disorder field theory dual to Ginzburg-Landau's order field theory has led to his discovery of the tricritical point in superconductors in 1982. By studying the limit of strongly bound electron pairs as the origin of high- T_c superconductivity he pointed out that pairing and phase decoherence may occur at different temperatures. In ^3He he found, together with Kazumi Maki who has written an article in this part, an interesting helical texture whose existence was subsequently confirmed experimentally. To explain the blue phase of cholesteric liquid crystals, Kleinert and Maki furthermore investigated icosahedral symmetries. Such structures were later discovered in sputtered aluminum and are now called quasicrystals. In between their joint calculations Maki especially enjoyed singing Italian operas with Kleinert, who knows a great number of them by heart.

Part V comprises articles on topological defects and their role in phase transitions as well as on fluctuating strings and membranes. Defects play an important role in crystal melting, superfluidity, superconductivity, cosmology, and elementary particle physics as described in Kleinert's above-mentioned monographs. Several new developments are discussed there. As far as strings and membranes are concerned, the key point of Kleinert's research empha-

sized the structural similarity between real membranes and the surfaces swept out by color-electric strings which hold quarks together. This led to the by now famous Polyakov-Kleinert action for curvature-dominated membranes and strings. Many field theoretic techniques could be successfully transferred between these physically quite different fields. In particular, a fundamental constant which rules the so-called Helfrich pressure exerted by fluctuating membranes was thus found analytically for the first time in good agreement with earlier Monte Carlo simulations.

The final Part VI deals with questions related to gravitation, cosmology, and astrophysics. Here Kleinert's research interests focused mainly around extending Einstein's gravitational theory in such a way that also the torsion of space-time can be taken into account. In close analogy to forces between defects in real crystals he developed a replacement and extension of Einstein's equivalence principle in the form of a non-holonomic mapping principle which permits deriving laws of Nature in geometries with curvature and torsion from those in flat space-time. In using non-holonomic mappings, Kleinert closed the circle between gravity, the theory of dislocations and disclinations, and his early path-integral solution of the hydrogen atom. In gravity, he also obtained interesting consequences for the early universe by deriving a gravitational action which differs from Einstein's at high curvature as a result of quantum fluctuations of elementary fields.

The comprehensive work of Hagen Kleinert is witnessed by his four books and more than 300 research papers, all accessible on his internet homepage

<http://www.physik.fu-berlin.de/~kleinert>.

His work cannot be fully reflected by the selected contributions in this *Festschrift*. It contains 65 articles arranged in each part in topical order and further linked together through an extensive index. We can only hope that in this way at least a flavor of Kleinert's diverse research interests and achievements becomes visible. We refrain from reproducing his current list of publications as it continues to grow rapidly and would be outdated at the time of printing.

We are grateful to all authors for their immediate agreement to contribute an article. Many of them emphasize Kleinert's stimulating influence on their research. His Italian-, Spanish-, French-, and certainly English-speaking visitors all have an easy time discussing with him since he converses with them in their own language. The number of former and present students, postdocs, collaborators, and friends of Hagen Kleinert is so large that we were unable

to ask all of them to contribute to this book. We apologize to all those we have not been able to contact. Financial support from the HSP III program of the German Ministry of Education and Research (BMBF) is appreciated. We thank Sieglinde Endrias for reading the manuscript and spotting many printing errors, Renate Schmidt and Lea Voigt for secretarial help as well as Ms. E.H. Chionh for the pleasant negotiations with the publishing company. Finally, we acknowledge the constant interest and valuable advice of Dr. Annemarie Kleinert. She was very helpful in the editing process and brought several former collaborators of her husband to our attention.

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